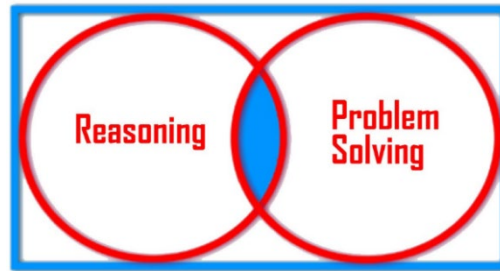# GCSE Foundation (5 – 1)

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Mathematical Reasoning Questions

(The Averages and Range) – Set 1

Solutions

The questions are repeated here for your convenience

1	John is taller than his brother Emmanuel.
	Their sister Alice is 161 cm tall.
	The mean height of the three siblings is 200 cm.
	Choose which statement is true
	   a) Both John and Emmanuel are shorter than Alice
	   b) Both John and Emmanuel are taller than Alice
	   c) Either John or Emmanuel must be taller than Alice
	   d) All the above could be true
	<u>**Solution**</u>
	The mean of a set of values is equal to all the values added together and the sum divided by how many there are.
	Since Alice's height (161 cm) is below the mean (200 cm), then at least one of John or Emmanuel must be taller than Alice for the mean height to be 200 cm
	Correct Answer: C **[1mark]**

2	What is the median of the numbers below?
	$$5^3, \quad 85, \quad 8^2, \quad 58^5, \quad 85^8$$
	Choose one answer
	   a) 5^3
	   b) 8^2
	   c) 85^8
	   d) 58^5
	<u>**Solution**</u>
	The median is the middle value when the numbers are arranged in order of size. (From smallest to largest)
	$$85, \quad 5^3, \quad \boxed{8^2,} \quad 58^5, \quad 85^8$$
	The middle value of the 5 numbers is the 3rd value, which is 8^2.
	Correct Answer: B **[2marks]**

|	

3	The expressions below are arranged in order of size $$x, \quad y, \quad 2x, \quad 4x, \quad 8y$$ Which expression represents their range? Choose one answer
	
	    a) $8x + y$
	    b) $9xy$
	    c) $\frac{8y+x}{2}$
	    d) $8y - x$ **<u>Solution</u>**
	
	The range of a set of values is the difference between the largest and smallest values
	
	Since the expressions are already in order of size, the range will be equal to $8y - x$
	
	Correct Answer: D **[1mark]**

4	The mean of the numbers 15, 20, 18, 19, 18 is the same as the mean of the numbers 38, p, p
	What is the value of p? Choose one answer.
	    a) 42
	    b) 21
	    c) 08
	    d) 38 **<u>Solution</u>**
	Mean = Sum ÷ how many
	$\frac{15+20+18+19+18}{5} = \frac{90}{5} = 18$
	The mean of the second set can be obtained as follows:
	$\frac{38 + p + p}{3}$
	Since the two sets have the same mean, then
	$\frac{38+2p}{3} = 18$ {multiply both sides by 3}
	$38 + 2p = 54$ {subtract 38 from both sides}
	$2p = 16$ {divide both sides by 2}
	$p = 8$ Correct Answer: C **[3marks]**

5	The card labelled P has increased the mean of the numbers below by 2. What is the number on the card labelled P?
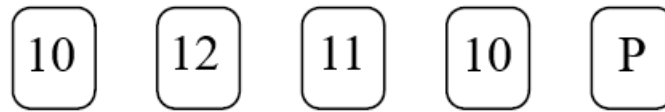
$$\boxed{10} \quad \boxed{12} \quad \boxed{11} \quad \boxed{10} \quad \boxed{P}$$

Choose one answer

   a) 20

   b) $20\frac{3}{4}$

   c) 9

   d) $12\frac{3}{4}$

<u>Solution</u>

Calculate the mean of the numbers without the number of card P

That is, $\frac{10+12+11+10}{4} = \frac{43}{4} = 10.75$

P increased the mean by 2, so the mean of all including the number on card P will be equal to $10.75 + 2 = 12.75$

Therefore, $\frac{10+12+11+10+P}{5} = \frac{43+P}{5} = 12.75$

$\frac{43+P}{5} = 12.75$ {multiply both sides by 5}

$43 + P = 63.75$ {subtract 43 from both sides}

$P = 20.75 = 20\frac{3}{4}$

Correct Answer: B   **[3marks]**

6 | Here is a frequency table

Expression	v	w	x	y	z
Frequency	5	6	10	9	1

Answer **True** or **False** to the statements below
   a) The mean will be an expression closer in value to x
   b) The median is x
   c) The range is $5 - 1 = 4$

Solution
   a) **False:** (It depends on how large the other values are in comparison to x)
   b) **True:** The sum of the frequencies is 31. The middle number will be the 16ᵗʰ value {That is $\frac{31}{2} = 15.5$ which rounds up to 16}. The 16ᵗʰ value will be an x
   c) **False** {You cannot calculate the range using the frequencies}

[3marks]

7 | The six numbers below are arranged in ascending order

   6,   10, x,   12,   13,   20

The median of the six numbers is 11.5
What is the value of x?
   a) 11.5
   b) 11
   c) 12
   d) 12.5

Solution

   6,   10, x,   12,   13,   20

The median value is the value in the middle of the set.
Since there are 6 numbers, the median will be halfway between x and 12.
Therefore, $\frac{x+12}{2} = 11.5$ {multiply both sides by 2}
$x + 12 = 23$ {subtract 12 from both sides}
$x = 11$

Correct Answer: B   **[2marks]**

8 Five positive whole numbers have the following features
    i) Their mean is 10
    ii) The smallest number is 4
    iii) The only number that occurs more than once is 6
    iv) The second largest of the numbers is 10
What is the range of the five numbers?
    a) 20
    b) 24
    c) 22
    d) 28

<u>Solution</u>

The mean is equal to the sum of the values in the set, divided by the number of values.

So, if the mean of 5 numbers is 10, the sum of the five numbers will be equal to $5 \times 10 = 50$

4 is the smallest number and 10 is the second largest
6 occurs at least twice.

Therefore, we have $4, 6, 6, 10, x$.
Where x is the unknown fifth number

$4 + 6 + 6 + 10 + x = 50$
$26 + x = 50$ {subtract 26 from both sides}
$x = 24$

Correct Answer: B   **[2marks]**

9	If $a = -4$, $b = 3a$ and $c = -4b$
	What is the mean of a, b and c? Choose one answer
	a) -8
	b) 0
	c) $10.\dot{6}$
	d) 8 **<u>Solution</u>**
	$a = -4$
	Therefore, $b = 3a = 3 \times -4 = -12$
	Therefore, $c = -4b = -4 \times -12 = 48$
	Mean $= \frac{-4+(-12)+48}{3} = \frac{-4-12+48}{3} = \frac{32}{3}$
	Correct Answer: C **[4marks]**
10	Here is a table showing the test scores of 15 students

Score	Frequency
10	2
11	6
12	8
13	4

Isla wants to calculate the mean score

<u>Here is Isla's Answer</u>

$10 \times 2 = 20$

$11 \times 6 = 66$

$12 \times 8 = 96$

$13 \times 4 = 52$

Total $= 234$

Mean $= 234 \div 4 = 58.5$

Is Isla correct? Explain your choice. (Do not work out the mean)

<u>Solution</u>

Isla's answer is wrong. This is because she divided the total number of marks by 4, when she should have divided it by 15, since there is a total of 15 students in the class.

There 4 different numbers but **not** a total of 4 numbers

{Or a similar explanation}

                                                           [1mark]