

FluidMaths

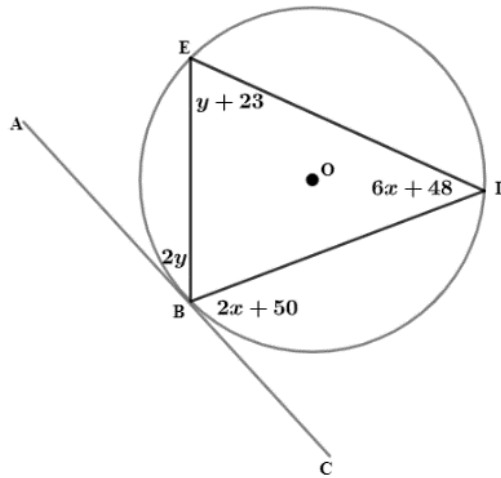
GCSE Mathematics

**Problem Solving aimed at students working
towards grade 9 or 8 (Solutions)**

**The grades and marks shown are for guidance
purposes only**

**The questions are repeated here for your
convenience**

- 1 In the diagram below, AC is a tangent to the circle with centre O



$$\text{Angle ABE} = 2y$$

$$\text{Angle CBD} = 2x + 50$$

$$\text{Angle BDE} = 6x + 48$$

$$\text{Angle BED} = y + 23$$

Show that $x:y = 1:11$

Solution

$$2y = 6x + 48$$

Therefore, $y = 3x + 24$ -----Equation 1
 {Using the alternate segment theorem} [1mark]

$$2x + 50 = y + 23$$

$y = 2x + 27$ -----Equation 2 [1mark]

$$\text{Thus, } 3x + 24 = 2x + 27$$

Therefore, $x = 3$ [1mark]

Substitute $x = 3$ into equation 1

$$y = 3 \times 3 + 24 = 33$$
 [1mark]

$$x:y = 3:33$$

Hence, $x:y = 1:11$ [1mark]

2 Given that $\cos p = \frac{5}{7}$

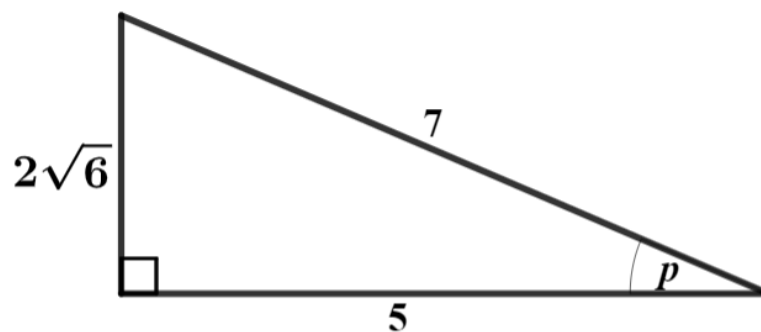
Calculate the exact value of $\sin p \times \tan p$.

Solution

Use Pythagoras theorem to calculate the third side of the triangle as follows

Note that $\cos p = \frac{A}{H}$

Therefore, the hypotenuse and adjacent of the triangle are 7 and 5 respectively



$$\sqrt{7^2 - 5^2} = \sqrt{49 - 25} = \sqrt{24} = 2\sqrt{6} \quad \text{[1mark]}$$

{Note that $\sqrt{24} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$ }

$$\sin p = \frac{2\sqrt{6}}{7} \quad \text{and} \quad \tan p = \frac{2\sqrt{6}}{5} \quad \text{[2marks]}$$

$$\text{Hence, } \sin p \times \tan p = \frac{2\sqrt{6}}{7} \times \frac{2\sqrt{6}}{5} = \frac{24}{35} \quad \text{[1mark]}$$

- 3** The probability that Rehan is selected for his team's football match is 0.5642
When Rehan plays, the probability that they win the match is 0.8821.
Calculate the probability that Rehan plays, and his team wins the match

Solution

This is a question on conditional probability.

$$\left\{ \text{Conditional Probability formula } P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)} \right\}$$

List the information given in the question

Let the probability that Rehan plays is $P(R) = 0.5642$

Then the probability that his team wins given Rehan plays will be $P(\text{Win given } R) = 0.8821$

Therefore, the probability that Rehan plays, and his team wins can be calculated as

$$P(R \text{ and Win}) = 0.8821 \times 0.5642 = 0.4977(4\text{dp})$$

[2marks]

- 4 A bag contains only red, blue and green counters.
The table below shows their probabilities

Counter	Red	Blue	Green
Probability	$2x$	x	

John picks a counter from the bag

He notes the colour and returns it

The probability that he picks a red followed by a blue counter is 0.1058

Calculate the probability of picking a green counter from the bag.

Solution

$$P(\text{Red and Blue}) = P(\text{Red}) \times P(\text{Blue})$$

{The Probability for independent events}

$$\text{Therefore, } P(\text{Red and Blue}) = 2x \times x = 2x^2$$

$$\text{Hence, } 2x^2 = 0.1058$$

[1mark]

$$\text{Therefore, } x = \sqrt{\frac{0.1058}{2}} = 0.23$$

[1mark]

$$\text{Therefore, } P(\text{Green}) = 1 - (2 \times 0.23 + 0.23)$$

$$= 1 - (0.69) = 0.31$$

[2marks]

5 Solve the simultaneous equations

$$5^{2x+y} = 125$$

$$3x + 2y = -7$$

Solution

We need to write the first equation as powers of 5

$$5^{2x+y} = 5^3 \quad \{\text{Note that } 125 = 5^3\}$$

Therefore, $2x + y = 3$ -----Equation 1

$$\text{So, } y = 3 - 2x \quad \text{[1mark]}$$

Now substitute $y = 3 - 2x$ into $3x + 2y = -7$

So, we have

$$3x + 2(3 - 2x) = -7 \quad \text{[1mark]}$$

$$3x + 6 - 4x = -7$$

$$-x = -13$$

$$\text{Therefore, } x = 13 \quad \text{[1mark]}$$

Substitute $x = 13$ into the equation $y = 3 - 2x$

$$\text{So, we have } y = 3 - 2 \times 13 = 3 - 26 = -23$$

[1mark]