

FluidMaths

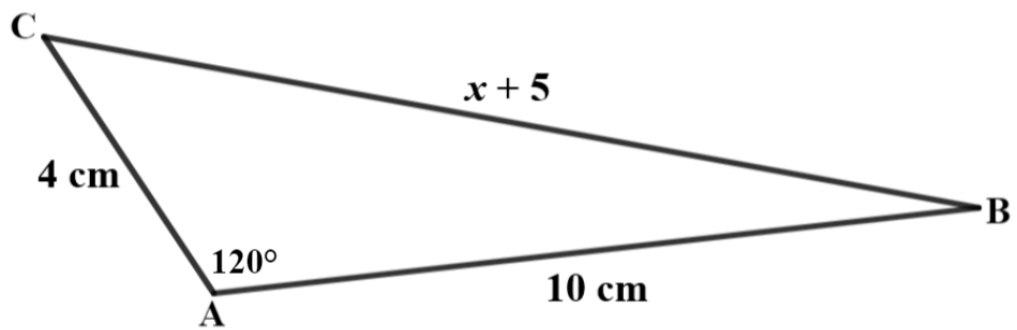
GCSE Mathematics (Grade 9-1)

Problem Solving -Trig Set 5
The cosine rule – Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 Here is triangle ABC



$$\text{Angle BAC} = 120^\circ$$

$$AB = 10 \text{ cm}$$

$$AC = 4 \text{ cm}$$

$$BC = x + 5$$

Calculate the true value of x

Give your answer to 1 decimal place

Solution

Apply the cosine rule: $\{a^2 = b^2 + c^2 - 2bc \times \cos A\}$

Therefore,

$$(x + 5)^2 = 4^2 + 10^2 - 2 \times 4 \times 10 \times \cos 120 \quad [1\text{mark}]$$

$$x^2 + 10x + 25 = 16 + 100 + 40 \quad [1\text{mark}]$$

$$x^2 + 10x + 25 = 156$$

$$\text{Therefore, we have } x^2 + 10x - 131 = 0 \quad [1\text{mark}]$$

$$\text{Use the quadratic formula: } \left\{ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right\}$$

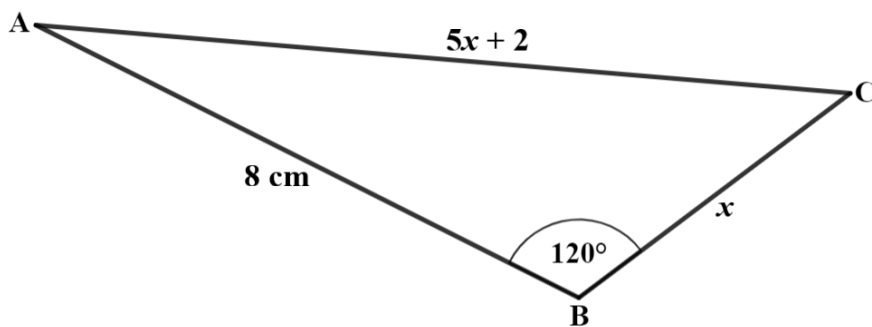
$$a = 1, \quad b = 10 \quad \text{and} \quad c = -131$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4 \times 1 \times -131}}{2 \times 1} \quad [1\text{mark}]$$

$$x = 7.49 \quad \text{or} \quad x = -17.5$$

$$\text{Hence, the true value of } x \text{ is } 7.5 \text{ (1dp)} \quad [1\text{mark}]$$

2 Triangle ABC is shown below



$$AB = 8 \text{ cm}$$

$$BC = x$$

$$AC = 5x + 2$$

$$\text{Angle } ABC = 120^\circ$$

$$\text{Show that } 2x^2 + x - 5 = 0$$

Solution

Apply the cosine rule: $\{a^2 = b^2 + c^2 - 2bc \times \cos A\}$

$$(5x + 2)^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 120 \quad [1\text{mark}]$$

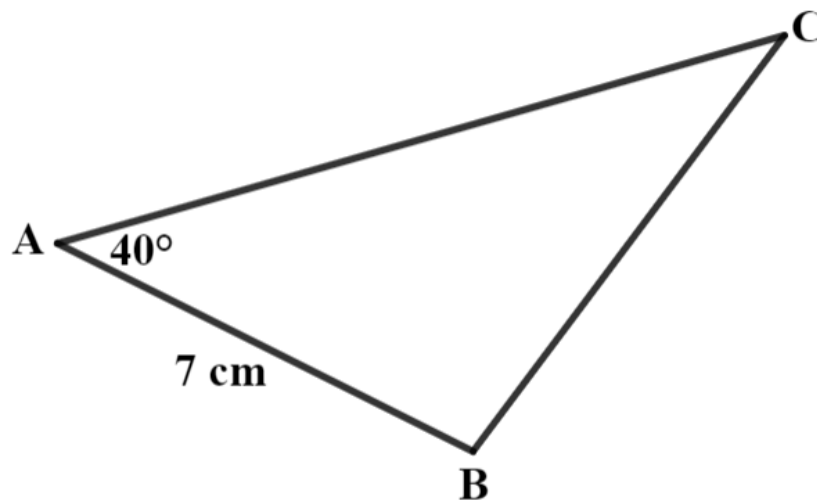
$$25x^2 + 20x + 4 = x^2 + 64 - 16x \times -\frac{1}{2} \quad [2\text{marks}]$$

$$\text{Therefore, } 25x^2 + 20x + 4 = x^2 + 8x + 64$$

$$24x^2 + 12x - 60 = 0 \quad \{\text{Divide both sides by 12}\} \quad [1\text{mark}]$$

$$2x^2 + x - 5 = 0 \quad [1\text{mark}]$$

3 ABC is a scalene triangle



The area of the triangle is 33 cm^2
 Calculate the perimeter of the triangle

Solution

The sine rule for the area of a triangle: $\left\{ \text{Area} = \frac{1}{2} ab \times \sin C \right\}$

$$\text{Therefore, } 33 = \frac{1}{2} \times 7 \times AC \times \sin 40 \quad [1\text{mark}]$$

$$66 = 4.50 \times AC$$

$$\text{Therefore, } AC = 14.7 \quad [1\text{mark}]$$

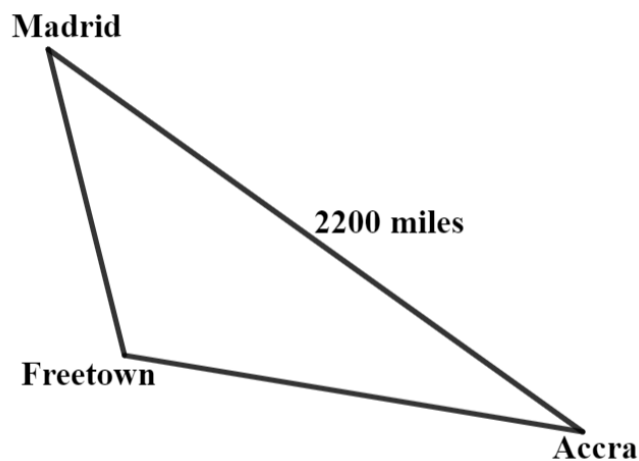
Use cosine rule: $\{a^2 = b^2 + c^2 - 2bc \cos A\}$

$$BC^2 = 7^2 + 14.7^2 - 2 \times 7 \times 14.7 \times \cos 40 \quad [1\text{mark}]$$

$$BC = \sqrt{107} = 10.3 \text{ cm}(1\text{dp}) \quad [1\text{mark}]$$

$$\text{Hence the perimeter of the triangle} = 7 + 14.7 + 10.3 = 32 \quad [1\text{mark}]$$

4 The diagram below shows the locations of three cities.



The bearing of Accra from Madrid is 158°

The bearing of Freetown from Madrid is 196°

The distance between Madrid and Accra is 2200 miles

A plane flying at a constant speed of 500 mph took 2 hours 15 minutes to Freetown from Madrid.

The plane is flying to Accra from Freetown at a constant speed of 350 mph.

Show that the plane will reach Accra after 4 hours.

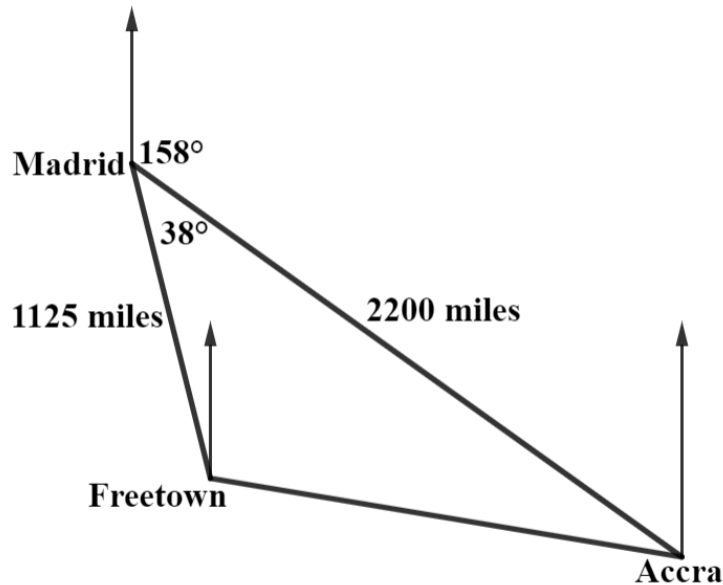
Solution

Distance = Speed \times Time

The distance between Freetown and Madrid is

$$500 \times 2.25 = 1125 \text{ miles} \quad [1\text{mark}]$$

{Refer to the diagram below}



Since the bearing of Freetown from Madrid is 196° , it means the angle opposite Freetown to Accra will be $196 - 158 = 38$

$$a^2 = 1125^2 + 2200^2 - 2 \times 1125 \times 2200 \times \cos 38 \quad [2\text{marks}]$$

$$a^2 = 2204971.77 \quad [1\text{mark}]$$

$$a = \sqrt{2204971.77} = 1484.9 \text{ miles} \quad [1\text{mark}]$$

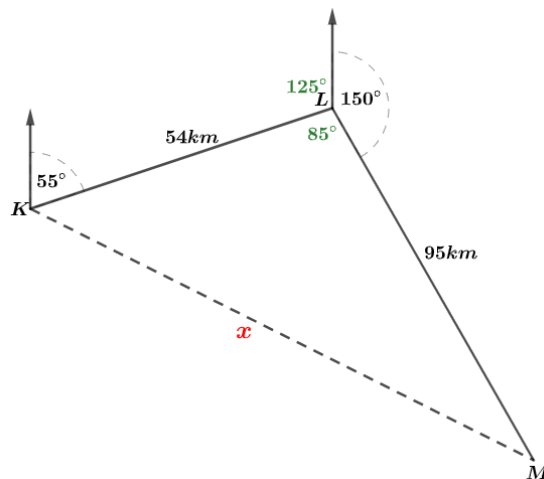
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{1484.9}{350} = 4.2 \text{ hrs(1dp)} \quad [1\text{mark}]$$

Hence, the plane took over 4 hours

- 5 Port L is 54 km and on a bearing of 055° from Port K
 Port M 95 km and is on a bearing of 150° from Port L
 Calculate the direct distance between ports M and K
 Give your answer to 3 significant figures

Solution

Make a sketch



Use the cosine rule: $\{a^2 = b^2 + c^2 - 2bc \times \cos A\}$

Let x be the direct distance between K and M

Then, $x^2 = 95^2 + 54^2 - 2 \times 95 \times 54 \times \cos 85$ [2marks]

$x^2 = 11046.8$ [1mark]

$x = 105 \text{ km}(3\text{sf})$ [1mark]