FluidMaths

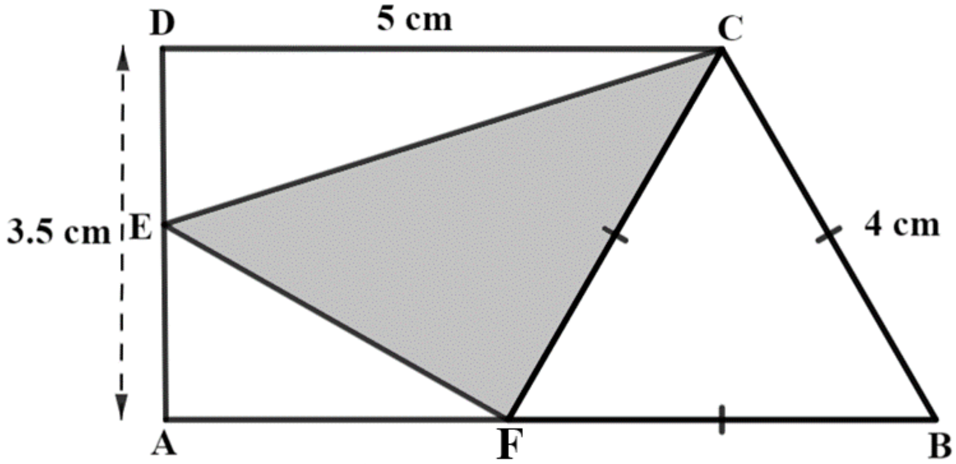
GCSE Mathematics (Grade 9-1)

Problem Solving -Trig Set 4
The sine rule – Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 ABCD is a trapezium.

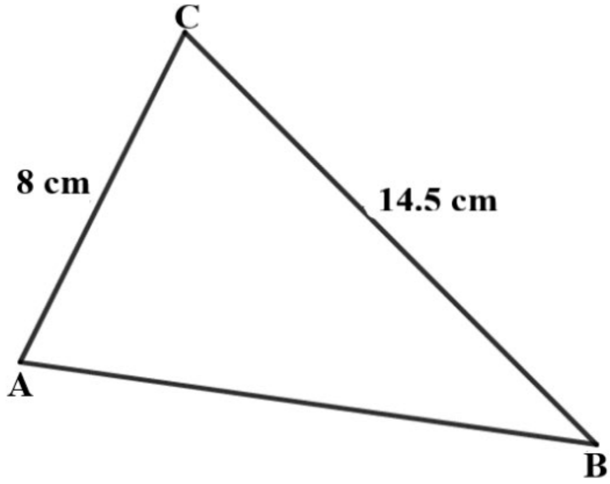


FBC is an equilateral triangle of side 4cm.
AD = 3.5 cm
CD = 5 cm
E is the midpoint on AD.
Calculate the area of triangle CEF
Give your answer to 2 decimal places.

Solution

E is the midpoint on AD. Therefore, DE = 1.75 cm
From triangle CDE, $CE^2 = 5^2 + 1.75^2$
$CE = \sqrt{5^2 + 1.75^2} = 5.3$ **[1mark]**
From triangle CDE, $\tan DCE = \frac{1.75}{5}$ **[1mark]**
Therefore, Angle $DCE = \tan^{-1}\left(\frac{1.75}{5}\right) = 19.3°$ **[1mark]**
So, Angle ECF = 360 – 90 – 90 – 60 – 60 – 19.3 = 40.7° **[1mark]**
The sine rule for the area of a triangle: $\left\{A = \frac{1}{2}ab \times \sin C\right\}$
Therefore, the area of triangle CEF is
$\frac{1}{2} \times 4 \times 5.3 \times \sin 40.7 = 6.91\text{cm}^2$ (2dp) **[1mark]**

2	Here is triangle ABC



AC = 8 cm
BC = 14.5 cm
The area of the triangle is 52.5 cm²
Calculate the size of angle ACB to the nearest degree

Solution

Use $\left\{\text{Area of a triangle} = \frac{1}{2}\text{absinC}\right\}$

Therefore, $52.5 = \frac{1}{2} \times 8 \times 14.5 \times \sin ACB$ **[1mark]**

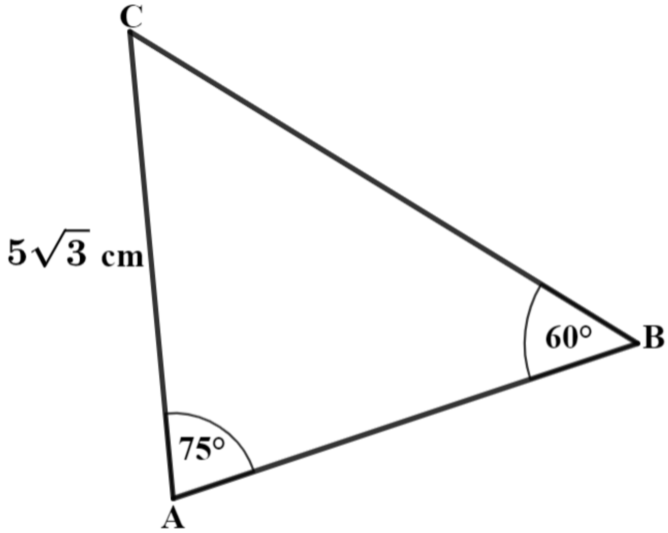
$105 = 116 \times \sin ACB$ **[1mark]**

$\sin ACB = \frac{105}{116}$

Angle $ACB = \sin^{-1}\left(\frac{105}{116}\right) = 64.8$ **[1mark]**

Therefore, Angle ACB is 65° to the nearest degree **[1mark]**

3	Triangle ABC is shown below



$AC = 5\sqrt{3}$ cm
Angle $CAB = 75°$
Angle $ABC = 60°$
Calculate the AB
Give your answer as a surd.

<u>Solution</u>

Apply the sine rule: $\left\{\frac{a}{\sin A} = \frac{b}{\sin B}\right\}$

Angle $ACB = 180 - 60 - 75 = 45°$

Therefore, $\frac{AB}{\sin 45} = \frac{5\sqrt{3}}{\sin 60}$

So, $AC = \frac{5\sqrt{3}}{\sin 60} \times \sin 45$ **[1mark]**

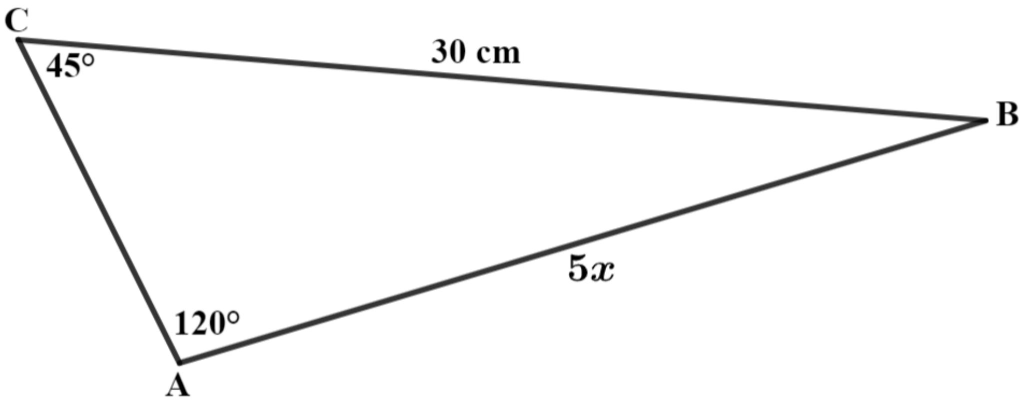
Therefore, $AC = 5\sqrt{3} \div \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$ **[2marks]**

{Note that $\sin 45 = \frac{\sqrt{2}}{2}$ and $\sin 60 = \frac{\sqrt{3}}{2}$}

$AC = 5\sqrt{3} \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{2}}{2}$ **[1mark]**

$AC = 5\sqrt{2}$ **[1mark]**

4	Here is triangle ABC



AB $= 5x$
BC $= 30$ cm
Angle CAB $= 120°$
Angle BCA $= 45°$
Show that $x = 2\sqrt{6}$

Solution

$\frac{5x}{\sin 45} = \frac{30}{\sin 120}$ **[1mark]**

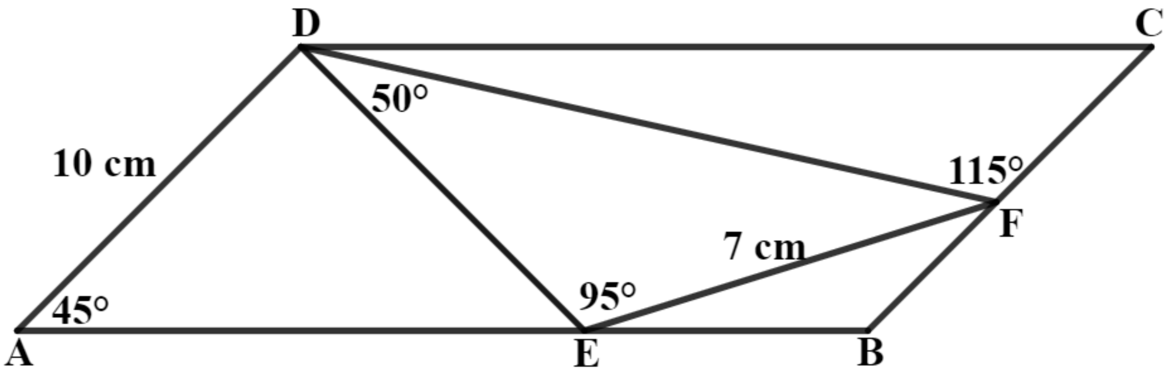
$5x \times \sin 120 = 30 \times \sin 45$

$5x = \frac{30 \sin 45}{\sin 120}$ **[1mark]**

$5x = 10\sqrt{6}$ **[1mark]**

$x = 2\sqrt{6}$ **[1mark]**

5 ABCD is a parallelogram



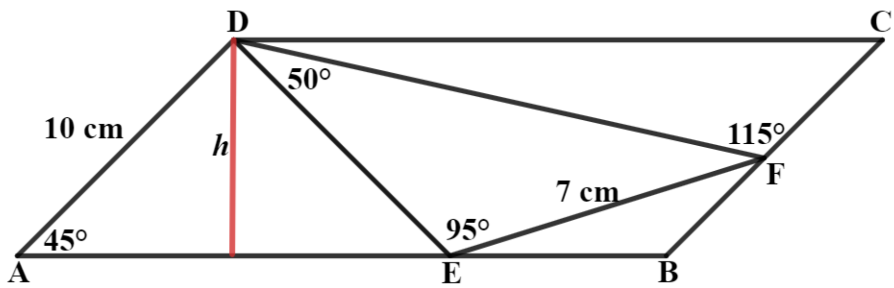
AD = 10 cm
EF = 7 cm
Angle DAE = 45°
Angle DEF = 95°
Angle DFC = 115°
Angle EDF = 50°

a) Show that the height of the parallelogram is $5\sqrt{2}$

b) Calculate the area of the parallelogram
Give your answer to 3 significant figures.

Solution

a) {Refer to the diagram below}



To find the height, consider triangle AED.

$\sin 45 = \frac{h}{10}$ **[1mark]**

Therefore, $h = 10 \times \sin 45 = 10 \times \frac{\sqrt{2}}{2} = 5\sqrt{2}$ **[2marks]**

b) Apply the sine rule for triangle DEF

$\frac{DF}{\sin 95} = \frac{7}{\sin 50}$ **[1mark]**

Therefore, $DF = \frac{7\times\sin 95}{\sin 50} = 9.1$ cm(1dp) **[1mark]**

Angle $DCF = 45°$ {Opposite angles in a parallelogram}

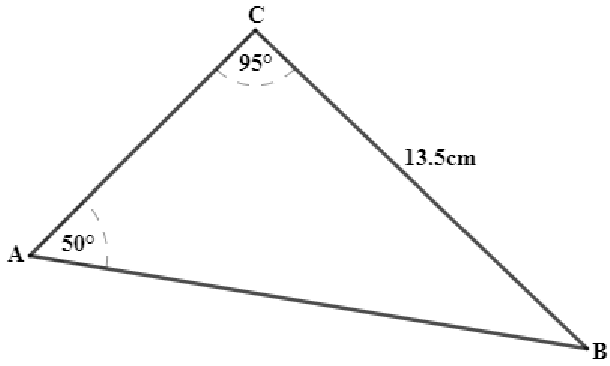
Apply the sine rule for triangle DCF $\frac{DC}{\sin 115} = \frac{9.1}{\sin 45}$

$DC = \frac{9.1\times\sin 115}{\sin 45} = 11.7$ **[1mark]**

The area of the parallelogram is

$11.7 \times 5\sqrt{2} = 82.7$ cm^2(3sf) **[1mark]**

6 Triangle ABC is shown below



BC = 13.5 cm
Angle ACB = 95°
Angle CAB = 50°
Calculate the perimeter of the triangle
Give your answer to 3 significant figures.

Solution

Apply the sine rule to find AB

$$\frac{AB}{\sin 95} = \frac{13.5}{\sin 50}$$

$$AB = \frac{13.5 \times \sin 95}{\sin 50} = 17.6 \text{ cm}$$ **[2marks]**

Also,

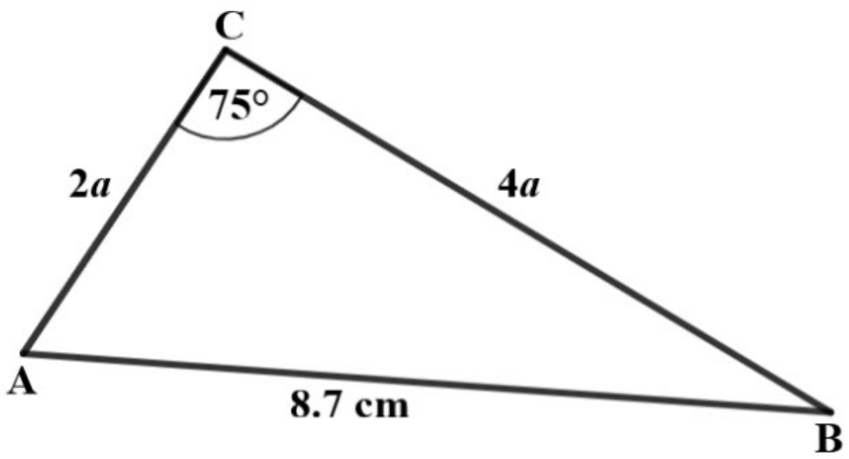
$$\frac{AC}{\sin 35} = \frac{13.5}{\sin 50}$$

$$AC = \frac{13.5 \times \sin 35}{\sin 50} = 10.1 \text{ cm}$$ **[2marks]**

Therefore, the perimeter of triangle ABC is

$$17.6 + 10.1 + 13.5 = 41.2 \text{ cm}$$ **[1mark]**

7 Triangle ABC is shown below



AB = 8.7 cm
Angle ACB = 75°
AC = $2a$
BC = $4a$
Given that the area of the triangle is 23.2 cm², calculate the perimeter of the triangle.
Give your answer to 1 decimal place.

Solution

$$\left\{\text{Area of a triangle} = \frac{1}{2}\text{ab} \times \sin \text{C}\right\}$$

Therefore, $23.2 = \frac{1}{2} \times 2a \times 4a \times \sin 75$ **[1mark]**

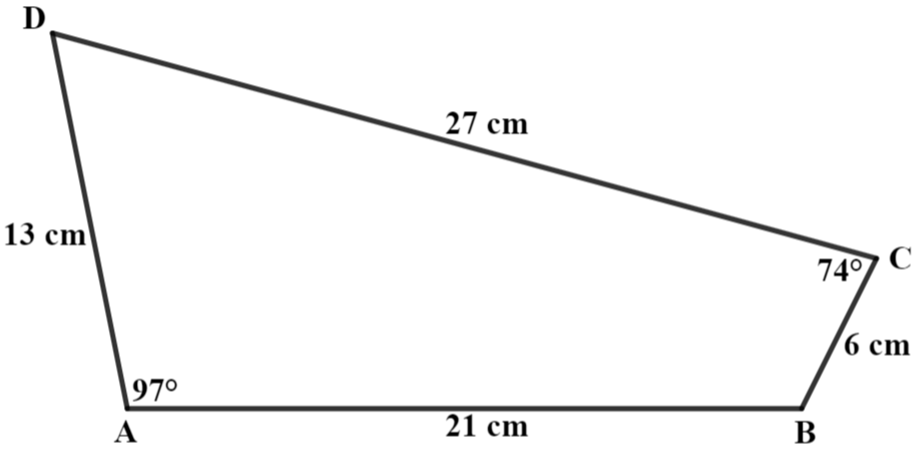
$23.2 = 4a^2 \times \sin 75$

$23.2 = 3.86a^2$ **[1mark]**

$a = \sqrt{\frac{23.2}{3.86}} = 2.45$ **[1mark]**

Therefore, the perimeter of triangle ABC will be
$2 \times 2.45 + 4 \times 2.45 + 8.7 = 23.4$ cm **[1mark]**

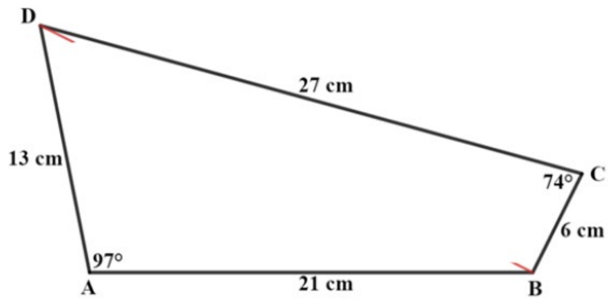
8 ABCD is a quadrilateral



AB = 21 cm
AD = 13 cm
CD = 27 cm
BC = 6 cm
Angle BAD = 97°
Angle BCD = 74°
Calculate the area of the quadrilateral.
Give your answer to 3 significant figures.

Solution

Split the quadrilateral into 2 triangles as shown

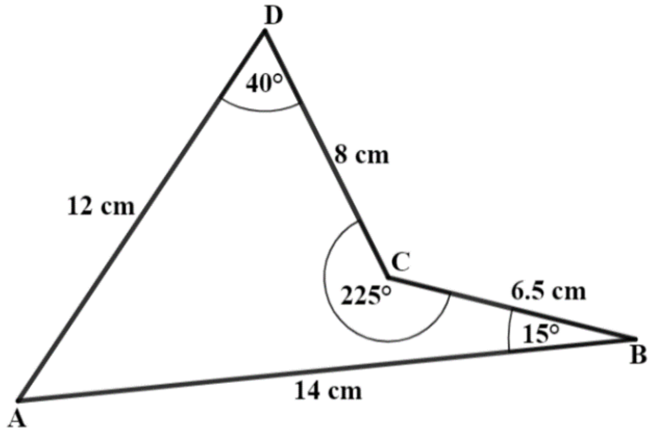


The area of triangle ABD=
$\frac{1}{2} \times 13 \times 21 \times \sin 97 = 135.5$
[1mark]

The area of BCD $= \frac{1}{2} \times 27 \times 6 \times \sin 74.1 = 77.9$ **[1mark]**

Therefore, the total area of the quadrilateral
$= 135.5 + 77.9 = 213\ \text{cm}^2$ (3sf) **[1mark]**

9 ABCD is a quadrilateral



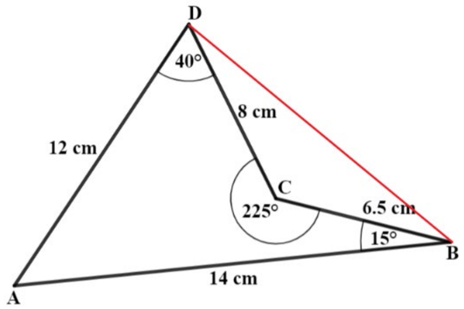
The reflex angle BCD = 225°

Calculate the area of the quadrilateral

Give your answer to 1 decimal place.

Solution

draw a line to connect B and D



Angle BAD is 360 – 255 – 40 – 15 = 80 **[1mark]**

Therefore, the area of triangle ABD will be

$\frac{1}{2} \times 12 \times 14 \times \sin 80 = 82.7$ **[1mark]**

Angle BCD = 360 – 255 = 135 **[1mark]**

The area of triangle BCD= $\frac{1}{2} \times 8 \times 6.5 \times \sin 135 = 13\sqrt{2}$ cm² **[1mark]**

Hence, the area of the quadrilateral ABCD

$= 82.7 - 13\sqrt{2} = 64.3 \text{ cm}^2$ (1dp) **[1mark]**