FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving -Trig 2 (SOHCAHTOA) – Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

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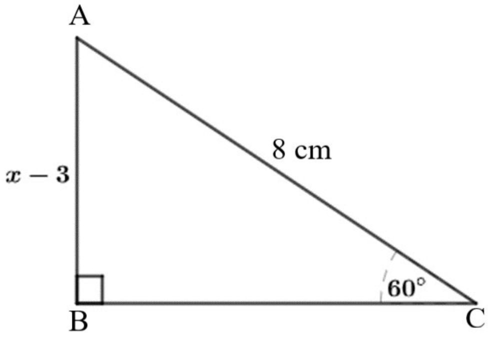
1 ABC is a right-angled triangle.



AC = 8 cm

AB = x – 3

Calculate the exact value of x.

Solution

Apply SOHCAHTOA

$\sin 60 = \frac{x-3}{8}$ **[1mark]**

So, $x - 3 = 8 \times \sin 60$

$x = 8 \times \sin 60 + 3$ **[1mark]**

$x = 8 \times \frac{\sqrt{3}}{2} + 3$ **[1mark]** {Note that $\sin 60 = \frac{\sqrt{3}}{2}$}

$x = 3 + 4\sqrt{3}$ **[1mark]**

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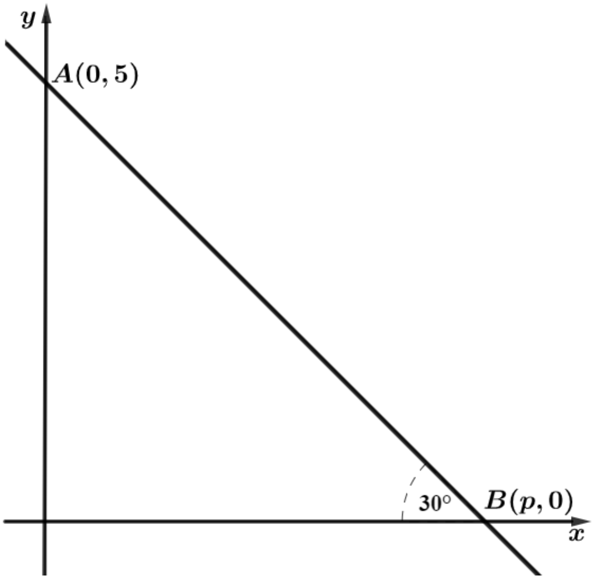
2 The graph of a straight line is shown below



The Line makes an angle of 30° with the x-axis

Point A has coordinates $(0, 5)$

Point B has coordinates $(p, 0)$

Show that $p = 5\sqrt{3}$.

Solution

Notice that the p is the distance between the x-intercept of the line and the origin (0, 0)

Apply SOHCAHTOA

The height of the triangle is 5

Therefore, $\tan 30 = \frac{5}{p}$ **[1mark]**

$p = \frac{5}{\tan 30} = \frac{5}{\frac{\sqrt{3}}{3}}$ **[1mark]**

{Note that the exact value of $\tan 30 = \frac{\sqrt{3}}{3}$}

Therefore, $p = \frac{15}{\sqrt{3}}$

$p = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$ **[2marks]**

Hence, $p = 5\sqrt{3}$

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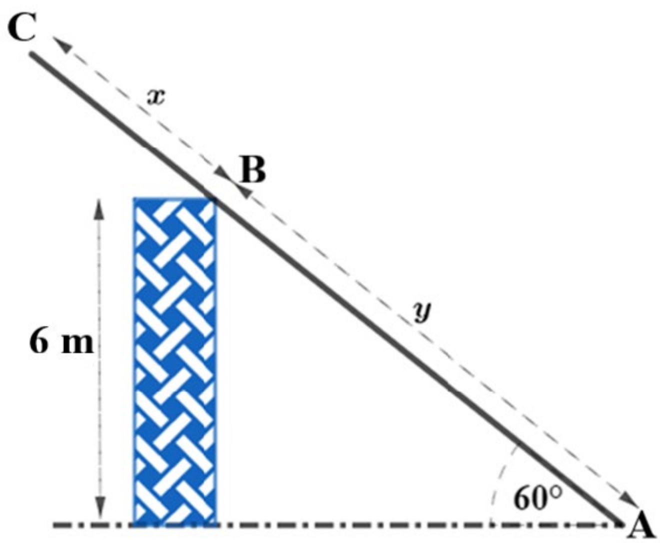
3	The diagram shows a stick AC leaned against a vertical wall The stick is inclined at an angle of 60° to the ground  B is the point where the stick touches the 6 m wall AB $= y$ BC $= x$ Given that the stick is $\sqrt{75}$ m long, Calculate the exact values of x and y **Solution** We can find the length of y using SOHCAHTOA $\sin 60 = \frac{6}{y}$ **[1mark]** $y = \frac{6}{\sin 60} = 4\sqrt{3}$ m **[1mark]** Therefore, $x = \sqrt{75} - 4\sqrt{3}$ $5\sqrt{3} - 4\sqrt{3} = \sqrt{3}$ m **[2marks]**

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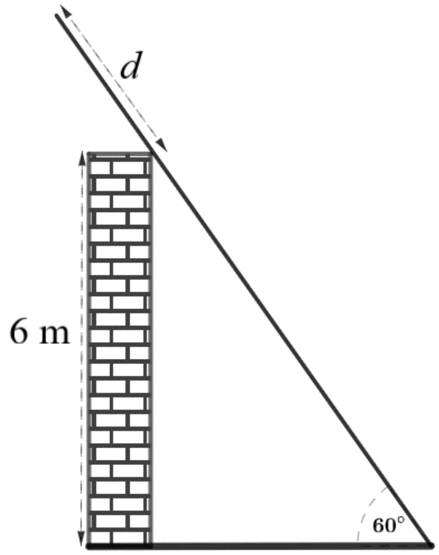
4 A 10 m stick is leaned against a 6 m wall such that, part of the stick goes over the wall as shown below.



The stick is inclined at an angle of 60° to the ground

Prove that $d = 10 - 4\sqrt{3}$

Solution

Use SOHCAHTOA

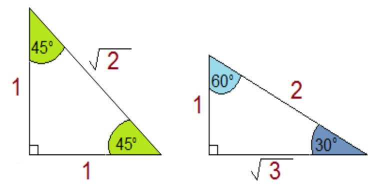
We have, $\sin 60 = \frac{6}{hyp}$ **[1mark]**

$\text{hyp} = \frac{6}{\sin 60}$

Therefore, $AB = \frac{6}{\frac{\sqrt{3}}{2}}$ **[1 mark]**

{Note that the exact value of $\sin 60 = \frac{\sqrt{3}}{2}$}

{Refer to the special triangles on the RHS}



$AB = \frac{12}{\sqrt{3}}$

$AB = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$ **[1mark]**

Therefore, $d = 10 - 4\sqrt{3}$ **[1mark]**

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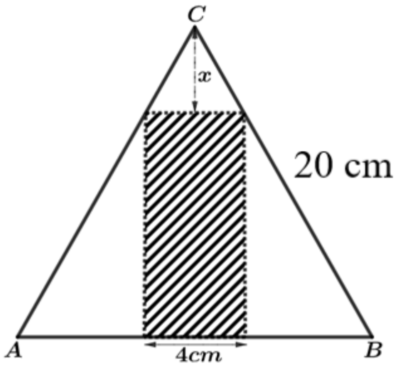
5 ABC is an equilateral triangle of side 20 cm.
A rectangular pattern is drawn inside the triangle as shown



The rectangular pattern is 4 cm wide
Calculate the exact value of x

Solution

Let the height of the triangle be h.

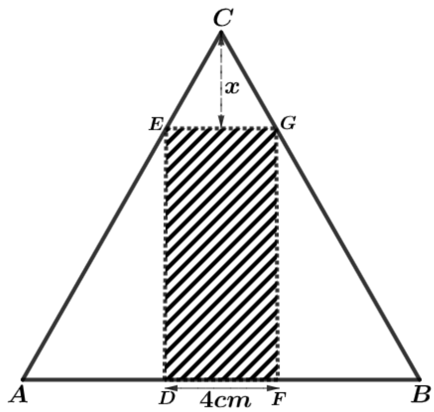
Then, $\sin 60 = \frac{h}{20}$

The triangle is equilateral therefore each angle is 60°

Therefore, $h = 20 \times \sin 60$

$h = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$ {Note that $\sin 60 = \frac{\sqrt{3}}{2}$} **[2marks]**

To find the height of the shaded pattern, consider triangle ADE where DE is the height of the pattern.



Since the shaded pattern is exactly in the middle of the triangle, it means that

AD = BF = 8 cm

{That is, 20 – 4 = 16 ÷ 2 = 8}

Therefore, $\tan 60 = \frac{DE}{8}$

So, $DE = 8 \times \sqrt{3} = 8\sqrt{3}$ **[2marks]**{Note that, $\tan 60 = \sqrt{3}$ }

Therefore, $x = 10\sqrt{3} - 8\sqrt{3} = 2\sqrt{3}$ **[1mark]**

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6	Given that $\sin x = \frac{2}{3}$ Calculate the exact value of $3\cos x + 5\tan x$

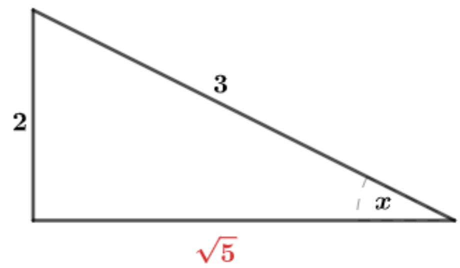
Solution

Draw a right-angled triangle to help visualise this problem

Since $\sin x = \frac{opp}{hyp}$

We can notice that, by using Pythagoras theorem, we can calculate the third side of the triangle as $\sqrt{3^2 - 2^2} = \sqrt{5}$

[1mark]



So, the triangle is shown below

Hence $\cos x = \frac{\sqrt{5}}{3}$ and $\tan x = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

[2marks]

Therefore $3\cos x + 5\tan x = 3 \times \frac{\sqrt{5}}{3} + 5 \times \frac{2\sqrt{5}}{5}$

[1mark]

$= \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$

[1mark]

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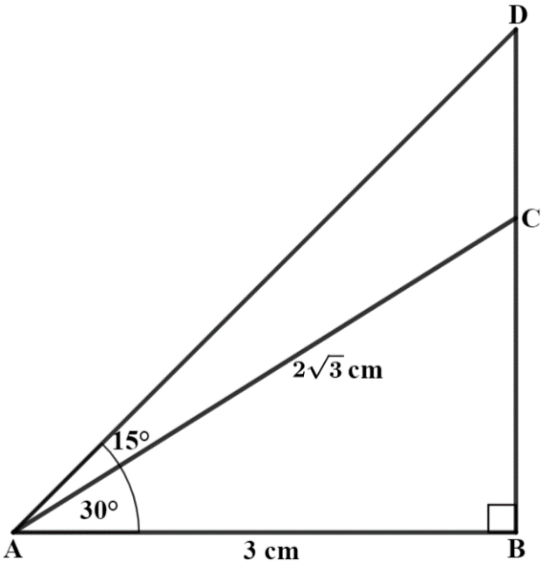
7 ABD is a right-angled triangle



AB = BD = 3cm

AC = $2\sqrt{3}$ cm

Angle BAC = 30°

Angle CAD = 15°

Show that $\sin 15 = \frac{\sqrt{6}-\sqrt{2}}{4}$

Solution

We need to find the side CD

We can then apply the sine rule to triangle ACD.

Note that angle CDA = 45°

Apply SOHCAHTOA to triangle ABC

$\tan 30 = \frac{BC}{3}$

$BC = 3 \times \tan 30$

$BC = \sqrt{3}$

Therefore, $CD = 3 - \sqrt{3}$ **[1mark]**

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	Now apply the sine rule to triangle ACD $\frac{\sin 15}{3-\sqrt{3}} = \frac{\sin 45}{2\sqrt{3}}$ **[1mark]** $\frac{\sin 15}{3-\sqrt{3}} = \frac{\frac{\sqrt{2}}{2}}{2\sqrt{3}}$ **[1mark]** $2\sqrt{3} \times \sin 15 = \frac{\sqrt{2}}{2} \times \left(3 - \sqrt{3}\right)$ $2\sqrt{3} \times \sin 15 = \frac{\sqrt{2}(3-\sqrt{3})}{2}$ $2\sqrt{3} \times \sin 15 = \frac{3\sqrt{2}-\sqrt{6}}{2}$ $\sin 15 = \frac{3\sqrt{2}-\sqrt{6}}{2} \div 2\sqrt{3}$ $\sin 15 = \frac{3\sqrt{2}-\sqrt{6}}{2} \times \frac{1}{2\sqrt{3}} = \frac{3\sqrt{2}-\sqrt{6}}{4\sqrt{3}}$ **[1mark]** $\sin 15 = \frac{3\sqrt{2}-\sqrt{6}}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{6}-\sqrt{18}}{12}$ **[1mark]** $\sin 15 = \frac{3\sqrt{6}-3\sqrt{2}}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$ **[1mark]** Hence, $\sin 15 = \frac{\sqrt{6}-\sqrt{2}}{4}$

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