

FluidMaths

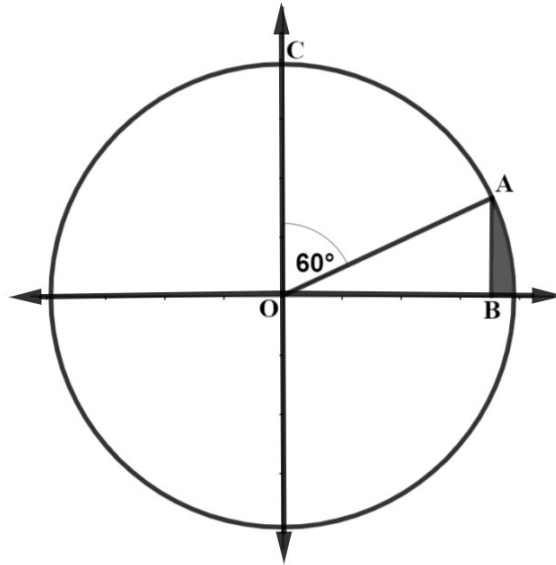
GCSE Mathematics (Grade 9-1)

Problem Solving
Trig Set 2 SOHCAHTOA
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 The diagram below shows a circle of radius of 3 cm



Angle AOC = 60°

Angle ABO = 90°

Calculate the shaded area to 1 decimal place

Solution

OA = 3 cm

Angle AOB = 30°

Calculate OB using SOHCAHTOA

$$\cos 30 = \frac{OB}{3}$$

OB = $3 \times \cos 30 = 2.60$ cm [1mark]

Calculate the area of triangle OBA

Use the sine rule {Area of a triangle = $\frac{1}{2}ab \times \sin C$ }

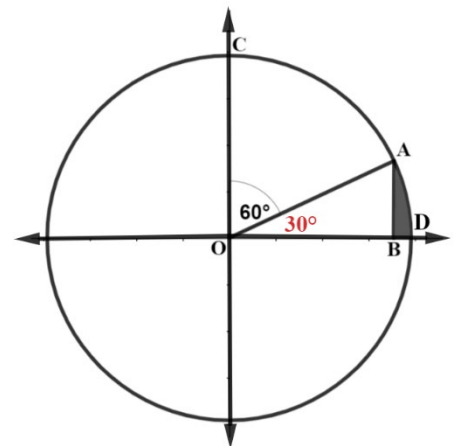
The area of triangle AOB = $\frac{1}{2} \times 3 \times 2.6 \times \sin 30 = 1.95$ cm²

[1mark]

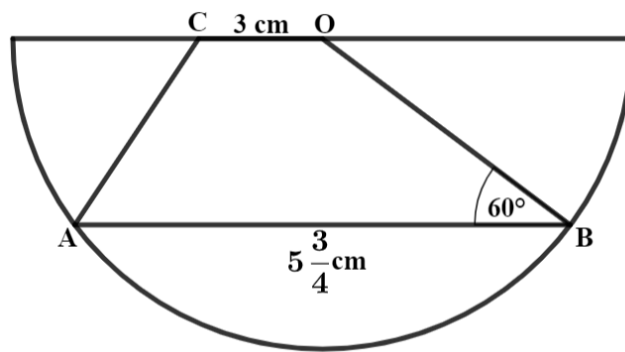
Area of the sector ODA = $\frac{30}{360} \times \pi \times 3^2 = 2.36$ cm² [1mark]

Therefore, the shaded are = $2.36 - 1.95 = 0.4$ cm² (1dp)

[1mark]



2 A trapezium ABOC is drawn inside a semi-circle as shown



The area of the trapezium is 26.25 cm^2

OB is the radius of the semi-circle.

$OC = 3 \text{ cm}$

$AB = 5\frac{3}{4} \text{ cm}$

Angle OBA = 60°

Calculate the area of the semi-circle to 3 significant figures

Solution

Find the height of the trapezium.

The area of a trapezium = $\frac{1}{2}(a + b)h$

Therefore, $26.25 = \frac{1}{2}\left(3 + 5\frac{3}{4}\right) \times h$ [1mark]

$$52.5 = 8\frac{3}{4}h$$

$$52.5 = 8.75h$$
 [1mark]

$$h = 52.5 \div 8.75 = 6$$
 [1mark]

Now we can find the radius of the semi-circle

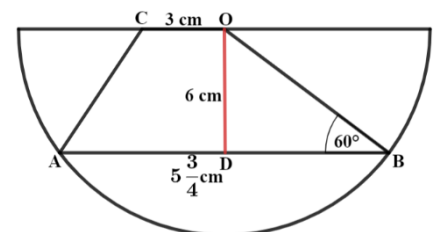
From triangle ODB {diagram on RHS}

$$\sin 60 = \frac{6}{OB}$$

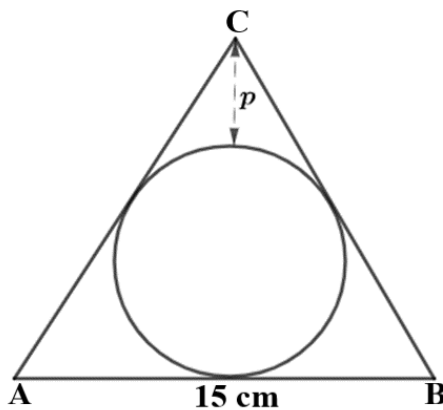
$$OB = \frac{6}{\sin 60} = 4\sqrt{3}$$
 [1mark]

The area of the semi-circle = $\pi(4\sqrt{3})^2 \div 2$

$$= 75.4 \text{ cm}^2 \text{ (3sf)}$$
 [1mark]



3 ABC is an equilateral triangle of side 15 cm



A circle of area $27\pi \text{ cm}^2$ is drawn inside the triangle
Calculate the value of p .

Give your answer to 1 decimal place

Solution

The height of the triangle is the diameter of the circle + p
{The area of a circle = πr^2 }

Therefore, $27\pi = \pi r^2$

$$r = \sqrt{27} = 3\sqrt{3} \quad \text{[1mark]}$$

Therefore, the diameter of the circle is

$$2 \times 3\sqrt{3} = 6\sqrt{3} \text{ [1mark]}$$

Use SOHCAHTOA to find the height of the triangle
ABC is equilateral.

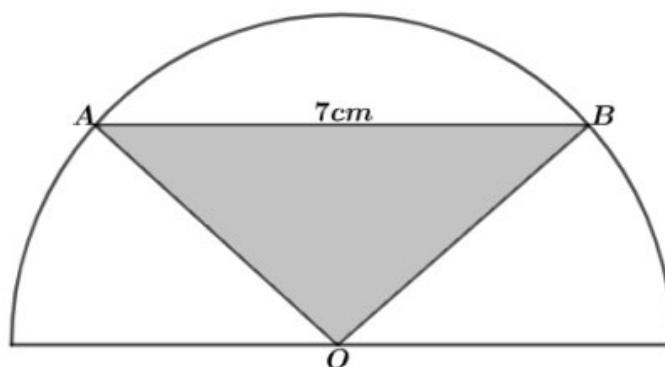
$$\text{Therefore, } \sin 60 = \frac{h}{15}$$

$$\text{Therefore, } h = 15 \times \sin 60 = \frac{15\sqrt{3}}{2} \quad \text{[1mark]}$$

$$p = \frac{15\sqrt{3}}{2} - 6\sqrt{3} = 2.598$$

$$\text{Hence, } p = 2.6 \text{ cm (1dp)} \quad \text{[1mark]}$$

4 A semi-circle of area 32 cm^2 and centre O is shown below



ABC is an isosceles triangle where $AO = BO$

$AB = 7 \text{ cm}$.

Calculate the area of the shaded triangle.

Give your answer to 2 decimal places.

Solution

O is the centre of the circle. Therefore, OA and OB are radii

$$\left\{ \text{The area of a semi circle} = \frac{\pi r^2}{2} \right\}$$

$$\text{Therefore, } 32 = \frac{\pi r^2}{2}$$

$$64 = \pi r^2$$

$$r = \sqrt{\frac{64}{\pi}} = 4.51 \text{ [1mark]}$$

From triangle OCB {Diagram on the RHS}

$$\sin COB = \frac{3.5}{4.51} \quad \text{[1mark]}$$

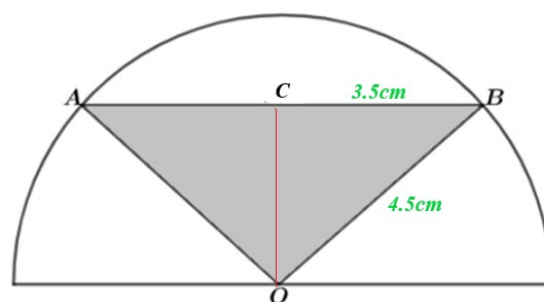
$$\text{Angle } COB = \sin^{-1} \left(\frac{3.5}{4.51} \right) = 50.9$$

Therefore, angle $AOB = 101.8^\circ$ [1mark]

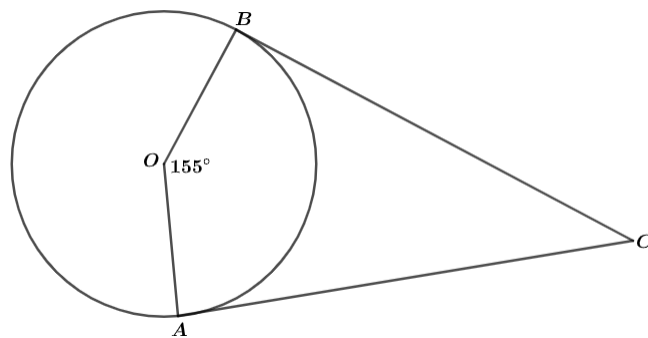
$$\left\{ \text{The area of a triangle} = \frac{1}{2} ab \times \sin C \right\}$$

Therefore, the area of triangle AOB is

$$\frac{1}{2} \times 4.51 \times 4.51 \times \sin 101.8 = 9.90 \text{ cm}^2 \text{ (2dp) [2marks]}$$



5 A circle of centre O is shown below



The area of the circle is 65 cm^2

ACBO is a quadrilateral

AC and BC are tangents to the circle

Calculate the distance OC

Give your answer to the nearest whole number.

Solution

Angle OAC is 90° {Angle between a radius and a tangent}

{The area of a circle: $A = \pi r^2$ }

Therefore, $65 = \pi r^2$

$$r = \sqrt{\frac{65}{\pi}} = 4.55 \quad \text{[1mark]}$$

Therefore, OA is 4.55 cm

Angle AOC is 77.5 because triangle AOC is congruent to triangle BOC.

$$\text{Therefore, } \cos 77.5 = \frac{4.55}{OC} \quad \text{[1mark]}$$

$$\text{Therefore, } OC = \frac{4.55}{\cos 77.5} = 21.0 \quad \text{[1mark]}$$

