

# FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving

Surface Area and Volume Set 2

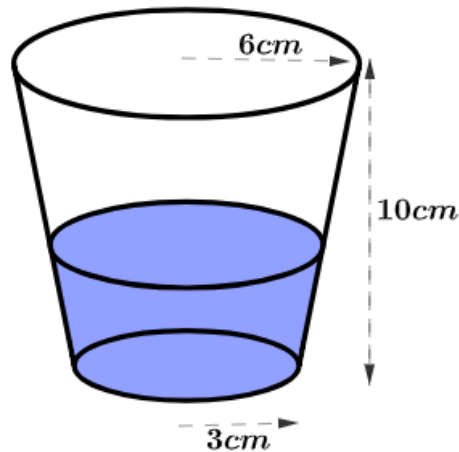
Spheres and Cones

Solutions

**The marks shown are for guidance purposes only**

**The questions are repeated here for your convenience**

1 A bucket of height 10 cm is filled with water.



The water takes up 30% of the capacity of the bucket.  
Calculate the volume of water in the bucket in terms of  $\pi$ .

$$\left\{ \text{Volume of Cone} = \frac{1}{3} \pi r^2 h \right\}$$

### Solution

We need to calculate the volume of the bucket first.

Notice that the bucket is a frustum of a cone.

Height of the cone from which the bucket is a frustum can be found as follows:

$6 \div 3 = 2$  gives the scale factor,

So, the height of the original cone is  $10 \times 2 = 20$  [1mark]

The volume of the original cone =  $\frac{1}{3} \times \pi \times 6^2 \times 20 = 240\pi$

[1mark]

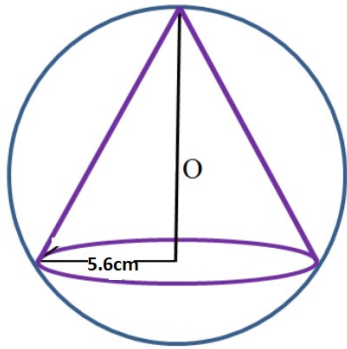
The volume of the cone which is removed to form

the bucket =  $\frac{1}{3} \times \pi \times 3^2 \times 10 = 30\pi$  [1mark]

The volume of bucket =  $240\pi - 30\pi = 210\pi$  [1mark]

30% of  $210\pi = \frac{30}{100} \times 210\pi = \frac{6300\pi}{100} = 63\pi$  [1mark]

- 2 The diagram below shows a sphere of center O. A cone is placed inside the sphere as shown.



$$\text{The volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{The volume of a cone} = \frac{1}{3}\pi r^2 h$$

The radius of the base of the cone is 5.6 cm.

The volume of the sphere is 7200 cm<sup>3</sup>.

- a) Calculate the radius of the sphere.

Give your answer to the nearest whole number

- b) Find the volume of the cone.

Give your answer to 3 significant figures

### Solution

- a) The volume of a sphere =  $\frac{4}{3}\pi r^3$

$$\text{Therefore, } 7200 = \frac{4}{3}\pi r^3$$

$$r^3 = \frac{7200 \times 3}{4\pi} \quad \text{[1mark]}$$

$$r = \sqrt[3]{\frac{7200 \times 3}{4\pi}} = 11.9788 \approx 12 \text{ cm [1mark]}$$

- b) Consider the diagram on the RHS

Apply Pythagoras theorem to triangle ACO

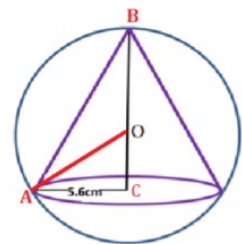
Note AO is a radius since O is the centre of

the sphere  $12^2 - 5.6^2 = (OC)^2$

$$OC = \sqrt{12^2 - 5.6^2} = 10.6 \quad \text{[1mark]}$$

The height of the cone is  $12 + 10.6 = 22.6 \text{ cm [1mark]}$

The volume =  $\frac{1}{3}\pi \times (5.6)^2 \times 22.6 = 742 \text{ cm}^3 \text{ (3sf) [1mark]}$



- 3** The surface area of a sphere is  $20 \text{ cm}^2$   
 Calculate the volume of the sphere  
 Give your answer to 2 significant figures.

$$\{\text{The surface area of a sphere} = 4\pi r^2\} \left\{ \text{The volume of a sphere} = \frac{4}{3}\pi r^3 \right\}$$

**Solution**

Calculate the radius of the sphere

$$\text{The surface area of a sphere} = 4\pi r^2$$

$$\text{Therefore, } 20 = 4\pi r^2$$

$$\frac{20}{4\pi} = r^2$$

$$\text{So, } r = \sqrt{\frac{20}{4\pi}} = 1.26 \text{ [2marks]}$$

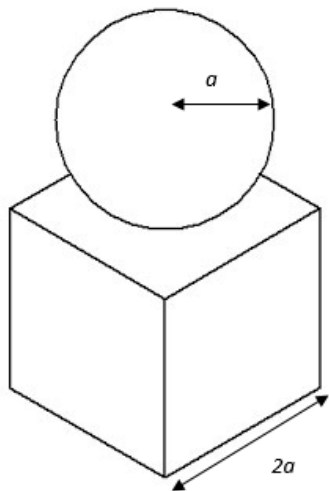
Now we can calculate the volume of the sphere

$$\text{The volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Therefore, The volume} = \frac{4}{3} \times \pi \times 1.26^3$$

$$V = 8.4 \text{ cm}^3 \text{ (2sf) [2marks]}$$

4 A sphere is placed on a cube as shown below



$$\left\{ \text{The volume of a sphere} = \frac{4}{3} \pi r^3 \right\}$$

The radius of the sphere is  $a$

The side length of the cube is  $2a$

Show that the volume  $V$  of the solid is equal to  $4a^3 \left( 2 + \frac{\pi}{3} \right)$

**Solution**

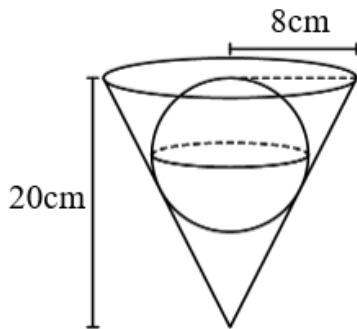
The volume of the Cube =  $2a \times 2a \times 2a = 8a^3$  [1mark]

The volume of the sphere =  $\frac{4}{3} \times \pi \times a^3 = \frac{4}{3} \pi a^3$  [1mark]

The volume of the solid =  $8a^3 + \frac{4}{3} \pi a^3$  {factorise}

Hence,  $V = 4a^3 \left( 2 + \frac{\pi}{3} \right)$  [2marks]

- 5 The diagram below shows a cone of height 20 cm  
The radius of the cone is 8 cm



$$\text{The volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{The volume of a cone} = \frac{1}{3}\pi r^2 h$$

The cone is filled with water.

When a sphere of radius  $r$  is placed inside the cone, some of the water is spilled.

Given that the amount of water left in the cone is  $550 \text{ cm}^3$ , calculate the radius of the sphere.

Give your answer to 3 significant figures.

### Solution

The volume of water that is spilled from the cone is equal to the volume of the sphere.

The volume of the cone is

$$\frac{1}{3} \times \pi \times 8^2 \times 20 = \frac{1280}{3}\pi = 1340 \quad [1\text{mark}]$$

$$\text{The volume of spilled water} = 1340 - 550 = 790 \quad [1\text{mark}]$$

So, the volume of the sphere is  $790 \text{ cm}^3$

$$\text{Therefore, } 790 = \frac{4}{3} \times \pi \times r^3 \quad [1\text{mark}]$$

$$r^3 = \frac{3 \times 790.4}{4\pi} \quad [1\text{mark}]$$

$$r = \sqrt[3]{\frac{2371.2}{4\pi}} = 5.74 \text{ cm (3sf)} \quad [1\text{mark}]$$

Therefore, the radius of the sphere is 5.74 cm