FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving
Surface Area and Volume Set 1
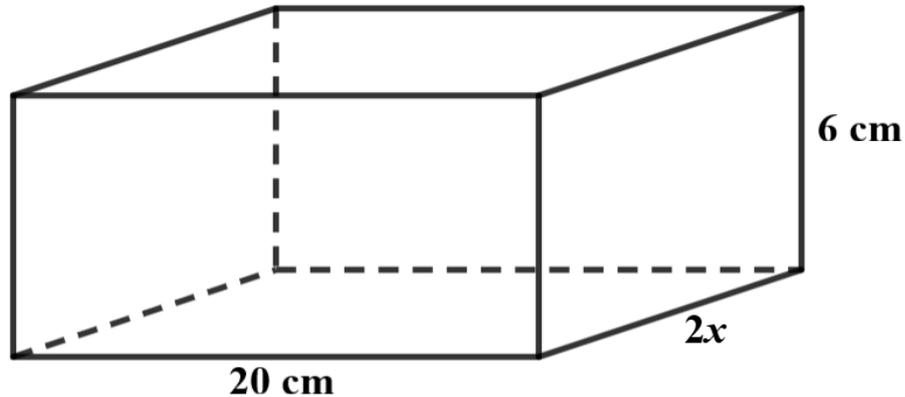### Prisms
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 | Here is a cuboid

The surface area of the cuboid is 500 cm²
Calculate the volume of the cuboid

Solution

The surface area in terms of x will be
$2(20 \times 6) + 2(20 \times 2x) + 2(6 \times 2x) = 240 + 80x + 24x$
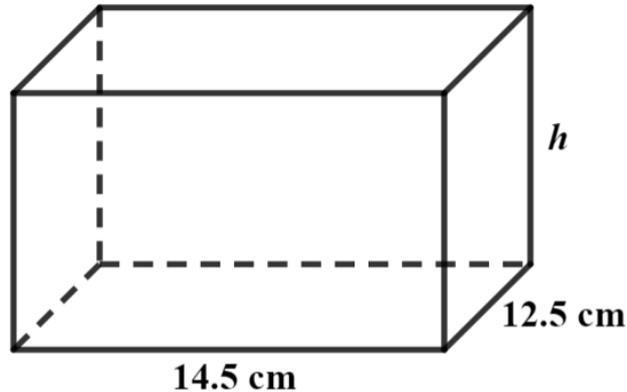Therefore, $104x + 240 = 500$ [2marks]

$104x = 260$
$x = 2.5$ [1mark]

Therefore, the volume of the cuboid will be
$20 \times 6 \times 5 = 600$ cm³ [1mark]

2 The volume of the cuboid below is 1812.50 cm³

The length of the cuboid is 14.5 cm
The width of the cuboid is 12.5 cm
The height of the cuboid is h
Calculate the surface area of the cuboid.

Solution

We need to calculate the height of the cuboid first
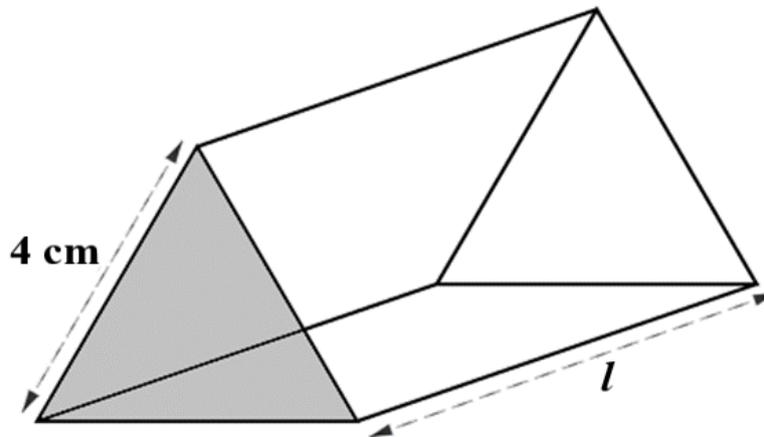$1812.50 = 14.5 \times 12.5 \times h$
$1812.50 = 181.25h$

$h = \frac{1812.5}{181.25} = 10$ **[1mark]**

SA $=(2 \times 12.5 \times 14.5) + (2 \times 12.5 \times 10) + (2 \times 14.5 \times 10)$
[2mark]

SA $= 362.5 + 250 + 290 = 902.5 \text{ cm}^2$ **[1mark]**

Hence the surface area of the cuboid is 902.5 cm²

3 Here is a triangular prism

The shaded cross-section is an equilateral triangle.
Each side of the triangle is 4 cm long
The area of the triangular cross-section is 14 cm²
The volume of the prism is 63cm³
The length of the prism is l
Calculate the surface area of the prism.

Solution

The volume of a Prism = Area of cross − section × Length
Therefore, $63 = 14 \times l$
So $l = 4.5$ **[1mark]**

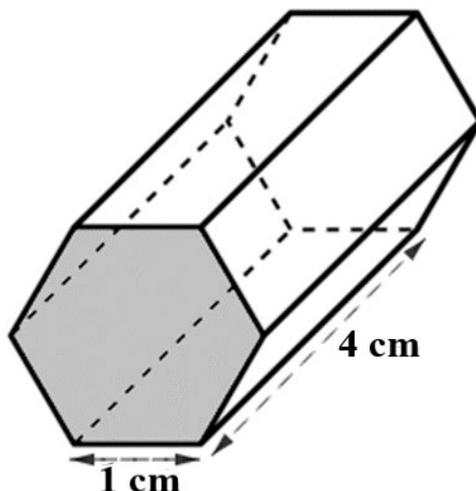Therefore, the area of each rectangular side is $4.5 \times 4 = 18$
Since there are 3 identical rectangular sides, we will have
$3 \times 18 = 54$ **[2marks]**

Hence, the surface area of the Prism will be
$54 + 14 + 14 = 82$ cm² **[1mark]**

4 Here is a prism

The cross-section of the prism is a regular hexagon.
The length of each side of the hexagon is 1 cm
The length of the prism is 4 cm
The surface area of the prism is 40 cm²
Calculate the volume of the prism

<u>Solution</u>

To find the volume, we need the area of the cross-section.
The area of each rectangular face is $4 \times 1 = 4$ **[1mark]**
The area of all six rectangular faces is $4 \times 6 = 24$ **[1mark]**
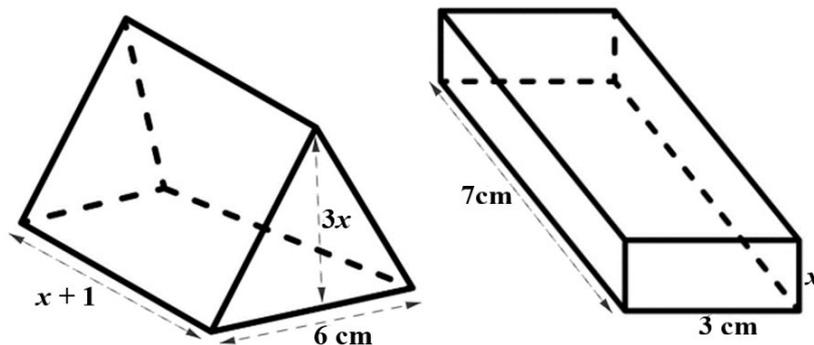Therefore, the area of the cross-section will be
$40 - 24 = 16 \text{ cm}^2$ **[1mark]**
However, there are two cross-sections at each end of the prism.
Therefore, each cross-section has an area of $16 \div 2 = 8$ **[1mark]**
Hence the volume of the prism is $8 \times 4 = 32 \text{ cm}^3$ **[1mark]**

5 | A triangular prism and a cuboid are shown below.

The length of the triangular prism is $x + 1$
The cross-section of the prism has a height of $3x$
and a base of 6 cm
The length and width of the cuboid are 7 cm and 3 cm
respectively. The height of the cuboid is x.
The prism and the cuboid have the same volume.
Calculate the value of x

Solution

The volume of the cuboid $= 3 \times x \times 7 = 21x$ [1mark]
The volume of the prism
$= \frac{1}{2} \times 6 \times 3x \times (x + 1) = 9x^2 + 9x$ [1mark]
The prism and the cuboid have the same volume
Therefore, $9x^2 + 9x = 21x$ [1mark]
$9x^2 + 9x - 21x = 0$
$9x^2 - 12x = 0$
$3x(3x - 4) = 0$ [1mark]
Therefore, $x = 0$ or $x = \frac{4}{3}$ [1mark]

Hence, the true value of x is $\frac{4}{3}$

6 Tank A and Tank B are shown below
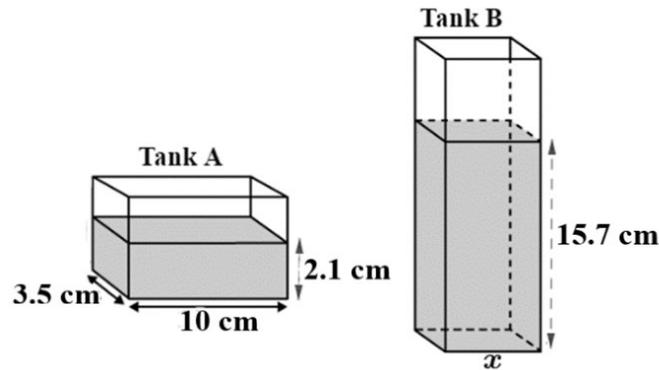
Tank A is filled with water to a depth of 2.1 cm
All the water from Tank A is then transferred into Tank B
without any waste
Tank B is a square-based prism.
The side of the square base is x
The water level in tank B is 15.7 cm
Calculate the value of x to 1 decimal place.

<u>Solution</u>

The volume of water in tank A
$3.5 \times 10 \times 2.1 = 73.5 \text{ cm}^3$ **[1mark]**

This is the same amount of water in tank B
The volume of water in tank B
$x \times x \times 15.7 = 15.7x^2$ **[1mark]**
Therefore, $15.7x^2 = 73.5$ **[1mark]**

$$x^2 = \frac{73.5}{15.7}$$

$$x = \sqrt{\frac{73.5}{15.7}} = 2.2 \text{ (1dp)} \quad \textbf{[1mark]}$$