FluidMaths

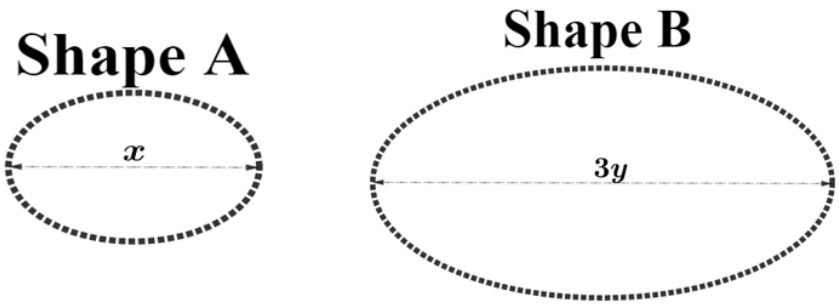
# GCSE Mathematics (Grade 9-1)

## Problem Solving
## Similar Shapes Set 2
## Area and Volume – Solutions

**The marks shown are for guidance purposes only**

**The questions are repeated here for your convenience**

**1** Two mathematically similar shapes are shown below.




The area of Shape A is 28 cm$^2$
The area of Shape B is 112 cm$^2$
Show that $y = \frac{2}{3}x$

**Solution**

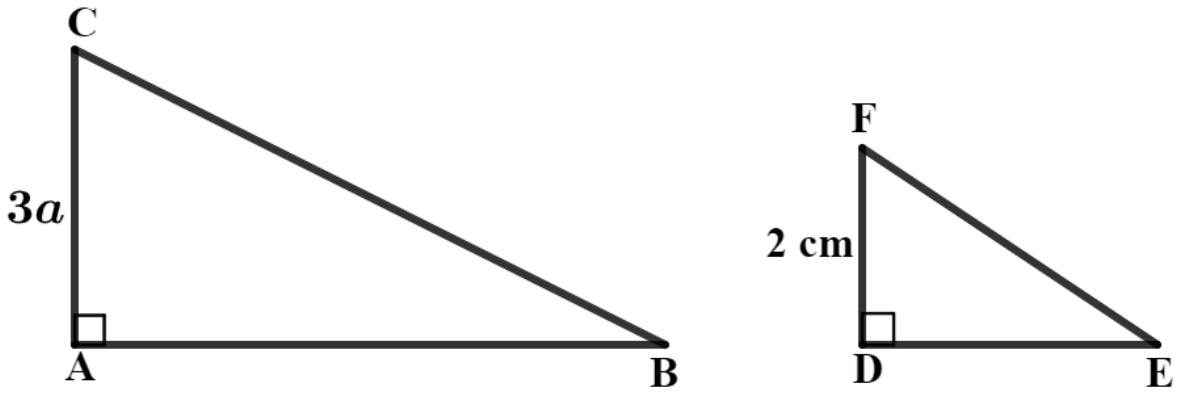
Area scale factor$= 112 \div 28 = 4$ **[1mark]**

Therefore, the scale factor for the lengths is $\sqrt{4} = 2$ **[1mark]**

So, $3y = 2 \times x$ **[1mark]**

Hence, $y = \frac{2}{3}x$ **[1mark]**

**2** ABC and DEF are mathematically similar triangles




$AC = 3a$

$DE = 2$ cm

The area of triangle ABC is 24 cm$^2$

Show that the area of triangle DEF is equal to $\frac{32}{3a^2}$

**Solution**

The linear scale factor $= \frac{3a}{2}$

Therefore, the area scale factor $= \frac{9a^2}{4}$ **[1mark]**

The area of triangle DEF will be equal to

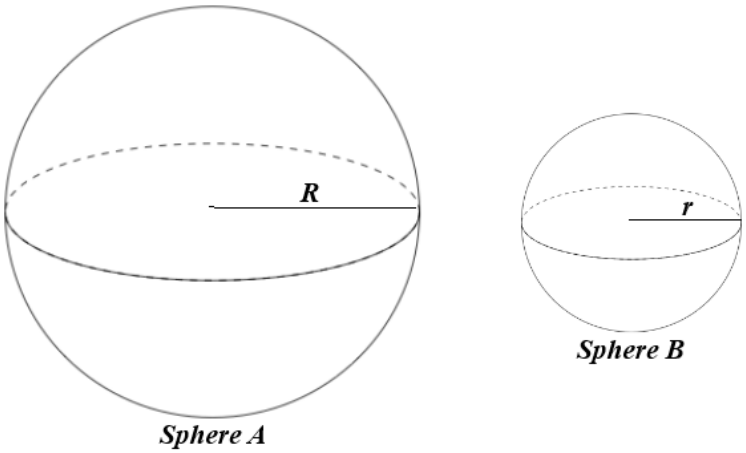
The area of triangle ABC ÷ by the area scale factor

Therefore, The area of triangle DEF $= 24 \div \frac{9a^2}{4}$ **[1mark]**

$= 24 \times \frac{4}{9a^2} = \frac{96}{9a^2}$ **[1mark]** {Simplify}

Hence, the area of triangle DEF $= \frac{32}{3a^2}$ **[1mark]**

**3** Sphere A and Sphere B are mathematically similar




The radius of sphere A is R
The radius of sphere B is $r$
The volume of sphere A is 360 $cm^3$
The volume of sphere B is 80 $cm^3$
Show that $R \approx 1.65r$

**Solution**

The volume scale factor, $360 \div 80 = 4.5$ **[1mark]**

To find the length scale factor we need to cube root the volume scale factor. {Note that $\text{Volume}_{sf} = (\text{Linear}_{sf})^3$}

Therefore, $\text{Linear}_{sf} = \sqrt[3]{4.5} = 1.65$ **[1mark]**
R = $r \times$ the linear scale factor
Hence, $R \approx 1.65r$ **[1mark]**

**4** Box A and Box B are mathematically similar
Box A is 40 cm wide
To make Box A, 36% less card is required than Box B.
How wide is Box B?

**Solution**

The surface area of Box A = 64% × surface area of Box B
Note that Since 36% less card is required, it means only 64% of the card needed for Box B is needed for Box A.

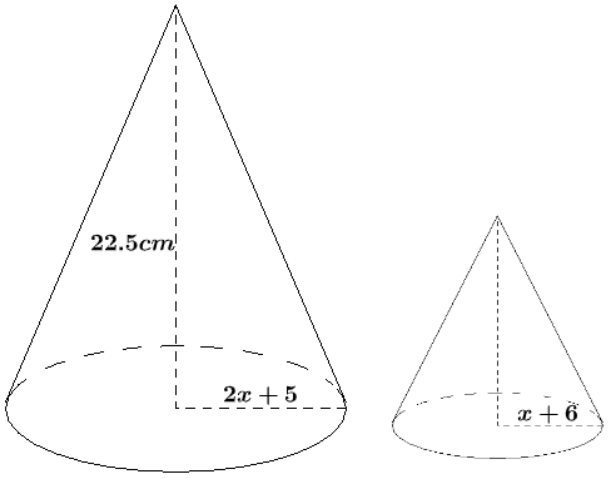
Therefore, SA of Box A = 0.64 × SA of Box B **[1mark]**
Thus, the area scale factor = 0.64

So, the scale factor for the lengths is $\sqrt{0.64} = 0.8$ **[1mark]**
{Note that Area scale factor = (Linear Scale Factor)$^2$}

So, 40 ÷ 0.8 = 50 cm **[1mark]**

Therefore, Box B is 50 cm wide

**5** Two mathematically similar cones are shown below.




The vertical height of the larger cone is 22.5 cm
The radius of the larger cone is $2x + 5$
The radius of the smaller cone is $x + 6$
The surface area of the larger cone is 69% more than the surface area of the smaller cone.

a) Calculate the value of $x$
b) Calculate the volume of the large cone.
Give your answer 2 decimal places.

**Solution**

a) Since the large cone has 69% more surface area than the smaller cone, it means that to obtain the surface area of the larger cone, we need to increase the surface area of the smaller cone by 69%.

Therefore, the area scale factor for the two cones in 1.69
{Note that a 69% increase will require a multiplier of 1.69}
Therefore the Linear Scale Factor $= \sqrt{1.69}$ **[1mark]**
{Note that Area Scale factor $=$ (Linear Scale Factor)$^2$}

Therefore $\sqrt{1.69} \times (x + 6) = 2x + 5$ **[1mark]**

$\frac{13}{10}(x + 6) = 2x + 5$ {Multiply both sides by 10}

$13x + 78 = 20x + 50$ **[1mark]**

$28 = 7x$

$x = 4$ **[1mark]**

b) The radius of the larger cone will be $2 \times 4 + 5 = 13$ **[1mark]**

Therefore, Volume $= \frac{1}{3} \times \pi \times 13^2 \times 22.5 = 3981.97(2dp)$ **[1mark]**

**6** Luke makes two mathematically similar ball bearings
The ratio of their radii is 3:2.
The bearings are made using the same metal plate.
Given that the smaller medal has a mass of 16.8g.
Find:
a) The mass of the larger ball bearing.
b) Show that the ratio of their masses is 27:8

**Solution**

The medals are made of the same material.
Therefore, they have the same density.
{Density = mass × volume}
Their radii are in the ratio 3:2
Therefore, the ratio of their volumes is 27:8 **[1mark]**
Let the mass of the larger bearing be m
Then, the density $= \frac{m}{27}$
The density of the smaller bearing is
density$= \frac{16.8}{8}$
Therefore, $\frac{m}{27} = \frac{16.8}{8}$ **[1mark]**

m$=27\times\frac{16.8}{8} = 56.7$ **[1mark]**
Therefore, the mass of the larger ball bearing is 56.7g

b) 56.7:16.8
567:168 (Divide both sides by 21)
27:8 **[1mark]**