FluidMaths

GCSE Mathematics (Grade 9-1)

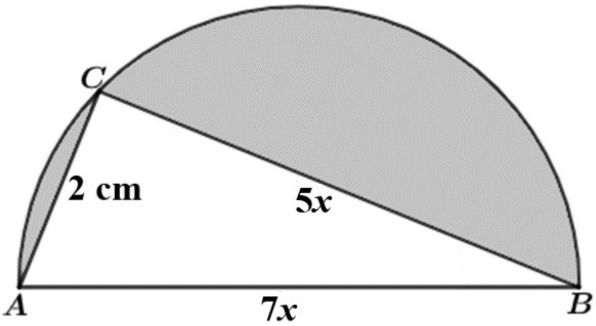
Problem Solving

Pythagoras Theorem Set 2 – Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 Triangle ABC is drawn inside a semi-circle



The diameter of the semi-circle is $AB = 7x$

$AC = 2$ cm and $BC = 5x$

Calculate the size of the shaded area

Give your answer to 2 decimal places.

Solution

{Circle Theorems: Angles in a semi-circle are right angles}

Therefore, triangle ABC is right-angled at C

Therefore, $(7x)^2 = (5x)^2 + (2)^2$ **[1mark]**

$49x^2 = 25x^2 + 4$

$24x^2 = 4$

$x = \sqrt{\frac{4}{24}} = 0.408$ **[1mark]**

So, the diameter of the semi-circle is

$AB = 7 \times 0.408 = 2.86$

Therefore, the radius $= 1.43$ **[1mark]**

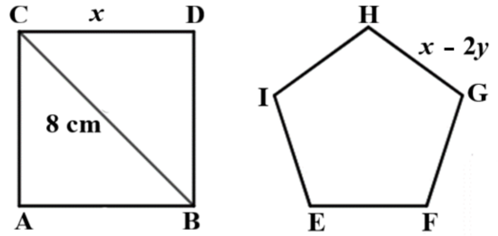
The area of the semi-circle

$= \frac{1}{2} \times \pi \times (1.43)^2 = 3.21$ **[1mark]**

The area of triangle ABC $= \frac{1}{2} \times 2 \times 5(0.408) = 2.04$ **[1mark]**

The shaded area = 3.21 – 2.04 = 1.17 cm² (2dp) **[1mark]**

2 ABCD is a square of side x
EFGHI is a regular pentagon of side $x - 2y$

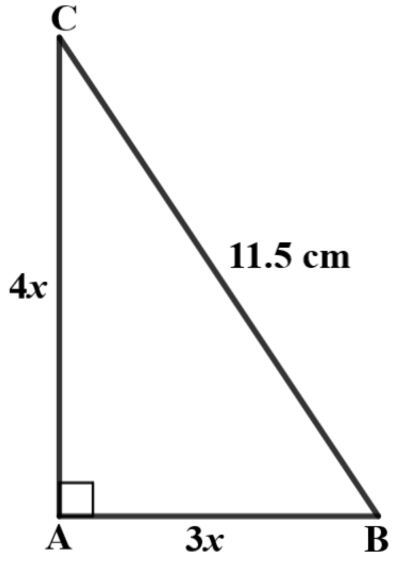


BC $= 8$ cm
The square and the pentagon have the same perimeter.
Show that $y = \frac{2\sqrt{2}}{5}$

Solution

From the square, $x^2 + x^2 = 8^2$
$2x^2 = 64$
$x^2 = 32$
$x = \sqrt{32} = 4\sqrt{2}$ **[2marks]**
The perimeter of the square is $4 \times 4\sqrt{2} = 16\sqrt{2}$ **[1mark]**
The perimeter of the pentagon is $5x - 10y$
The square and the pentagon have the same perimeter.
Therefore, $5x - 10y = 16\sqrt{2}$
Substitute $x = 4\sqrt{2}$ into $5x - 10y = 16\sqrt{2}$
$5(4\sqrt{2}) - 10y = 16\sqrt{2}$ **[1mark]**
$20\sqrt{2} - 10y = 16\sqrt{2}$
$10y = 20\sqrt{2} - 16\sqrt{2} = 4\sqrt{2}$
So, $10y = 4\sqrt{2}$
$y = \frac{4\sqrt{2}}{10}$ **[1mark]**
Hence, $y = \frac{2\sqrt{2}}{5}$ **[1mark]**

3 ABC is a right-angled triangle



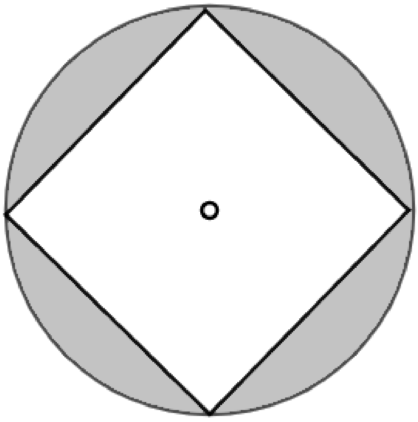
AB $= 3x$
AC $= 4x$
BC $= 11.5$ cm
Calculate the value of x to 1 decimal place

Solution

Apply Pythagoras theorem
$(4x)^2 + (3x)^2 = (11.5)^2$ **[1mark]**

$16x^2 + 9x^2 = 132.25$ **[2marks]**
$25x^2 = 132.25$
$x^2 = 5.29$
$x = \sqrt{5.29} = 2.3$ [2**marks**]

4 A square of area 256 cm^2 is drawn such that, its vertices lie on the circumference of a circle with centre O.



Show that the area of the shaded region can be written as $128(\pi - 2)$ cm^2

Solution

The shaded area = area of the circle – area of the square

The side length of the square $= \sqrt{256} = 16$ cm **[1mark]**

Let D be the diagonal of the square. Then, $D^2 = 16^2 + 16^2$

Therefore, $D = \sqrt{256 + 256} = \sqrt{512} = 16\sqrt{2}$ **[1mark]**

Therefore, the radius of the circle will be $8\sqrt{2}$ **[1mark]**

{The area of a circle $= \pi r^2$}

therefore, the area of the circle $= \pi \times (8\sqrt{2})^2$

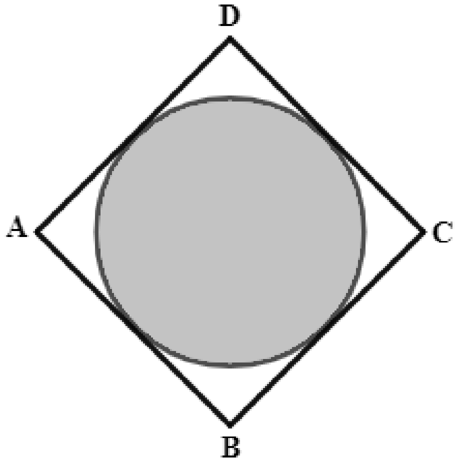
$= \pi \times 8\sqrt{2} \times 8\sqrt{2} = 64 \times 2 \times \pi$ **[1mark]**

$= 128\pi$ cm^2 **[1mark]**

The area of the shaded region$= 128\pi - 256$

$128(\pi - 2)$ cm^2 **[1mark]**

5	A circle is drawn inside the square ABCD as shown below



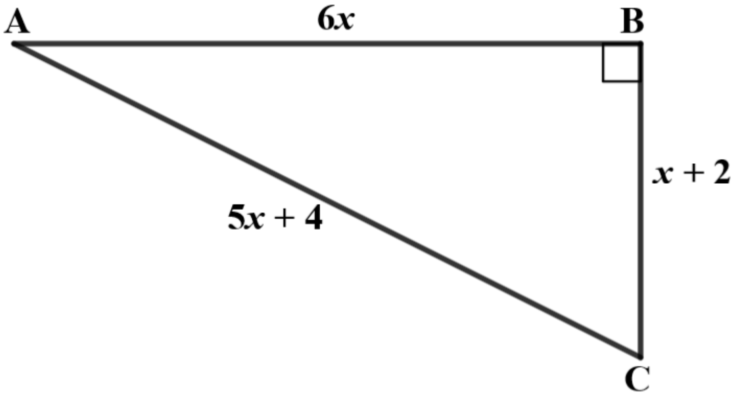
The sides of the square are tangents to the circle.
Given that AC = 10 cm,
show that the exact area of the circle is 12.5π cm²

Solution

Let the side of the square be x.
Then, $x^2 + x^2 = 10^2$
$2x^2 = 100$
$x^2 = 50$
$x = \sqrt{50} = 5\sqrt{2}$ **[2marks]**
Therefore, the diameter of the circle is $5\sqrt{2}$
Hence, the radius of the circle is $\frac{5\sqrt{2}}{2} = 2.5\sqrt{2}$

Therefore, the area of the circle is
Area $= \left(2.5\sqrt{2}\right)^2 \times \pi$ **[1mark]**
$\left(2.5\sqrt{2}\right)^2 = 2.5\sqrt{2} \times 2.5\sqrt{2} = 6.25 \times 2 = 12.5$
Hence, the area of the circle $= 12.5 \times \pi = 12.5\pi$ cm^2
[2marks]

6 ABC is a right-angled triangle



AB $= 6x$
AC $= 5x + 4$
BC $= x + 2$
Calculate the true value of x to 3 significant figures

<u>Solution</u>

Apply Pythagoras theorem
$(6x)^2 + (x+2)^2 = (5x+4)^2$ **[1mark]**
$36x^2 + x^2 + 4x + 4 = 25x^2 + 40x + 16$
$37x^2 - 25x^2 + 4x - 40x + 4 - 16 = 0$
$8x^2 - 36x - 12 = 0$ **[1mark]**
$2x^2 - 9x - 6 = 0$ **[1mark]**
Use the quadratic formula: $\left\{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right\}$
$a = 2; b = -9$ and $c = -6$
$x = \frac{--9 \pm \sqrt{(-9)^2 - 4 \times 2 \times -6}}{2 \times 2}$
$x = 5.09$ or -0.589 **[1mark]**

Therefore, the true value of $x = 5.09$ (3sf) **[1mark]**

7	Point A has coordinates $(a, 2)$, where $a > 1$ Point B has coordinates $(7, 5)$ The distance between A and B is $3\sqrt{5}$ What is the value of a? **Solution** Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $3\sqrt{5} = \sqrt{(a-7)^2 + (5-2)^2}$ **[1mark]** $(3\sqrt{5})^2 = (a-7)^2 + (5-2)^2$ **[1mark]** $45 = a^2 - 14a + 49 + 9$ $a^2 - 14a + 13 = 0$ {Factorise} **[1mark]** $(a-1)(a-13) = 0$ **[1mark]** Therefore, $a = 1$ or 13 But $a > 1$, hence $a = 13$ **[1mark]**