FluidMaths

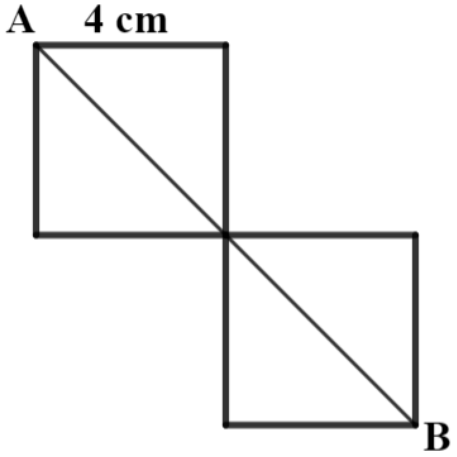
# GCSE Mathematics (Grade 9-1)

## Problem Solving
## Pythagoras Theorem Set 1
## Solutions

**The marks shown are for guidance purposes only**

**The questions are repeated here for your convenience**

**1** Two identical squares of side 4 cm are attached at a corner as shown below




Calculate the exact length of line AB
Give your answer in the form $a\sqrt{b}$

## Solution

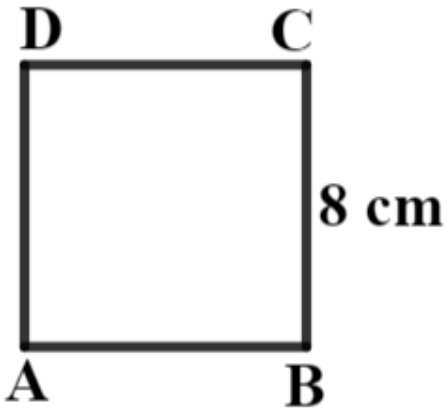
Apply Pythagoras theorem to find the diagonal of one

$4^2 + 4^2 = d^2$ **[1mark]**
where $d$ is the diagonal of one of the squares
$32 = d^2$ **[1mark]**

Therefore, $d = \sqrt{32} = 4\sqrt{2}$ **[1mark]**

Hence, the length of line AB $= 2 \times 4\sqrt{2} = 8\sqrt{2}$ **[1mark]**

**2** The diagram below shows a square ABCD of side 8 cm




Calculate the exact length of the diagonal AC
Give your answer as a simplified surd

**Solution**

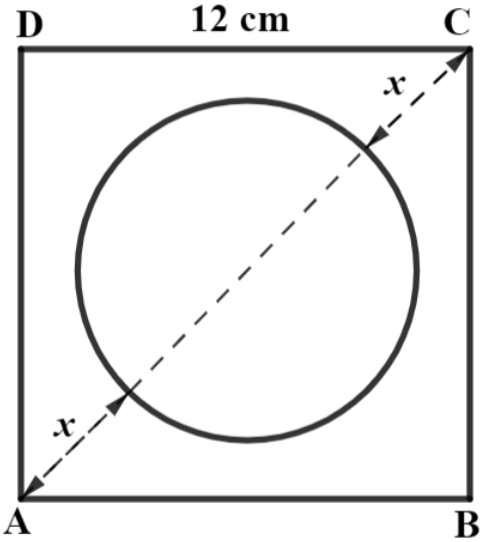
Apply Pythagoras theorem: $8^2 + 8^2 = AC^2$ **[1mark]**

$64 + 64 = AC^2$

$128 = AC^2$

Therefore, $AC = \sqrt{128} = 8\sqrt{2}$ **[1mark]**

**3** ABCD is a square of side 12 cm

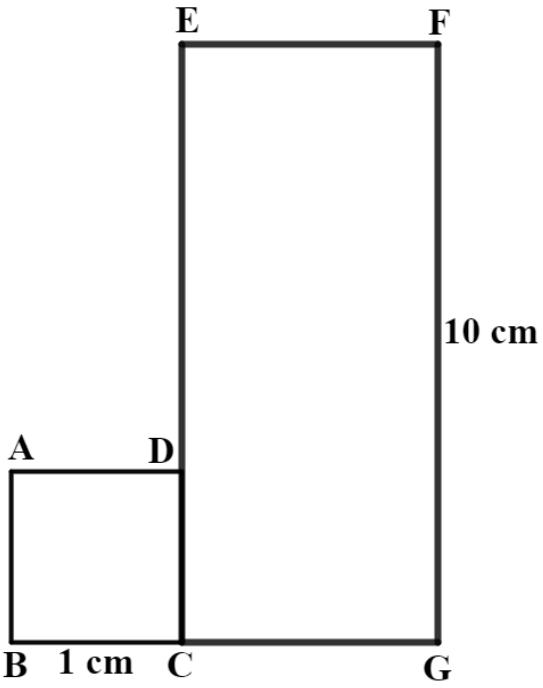



A circle of diameter 10 cm is drawn inside the square
The distance between the circumference of the circle and each vertex of the square is $x$
Calculate the value of $x$
Give your answer to 1 decimal place.

**Solution**

Apply the Pythagoras theorem to find the length of diagonal of the square: $12^2 + 12^2 = d^2$
where $d$ is the diagonal of the square
$288 = d^2$ **[1mark]**
$d = \sqrt{288} = 12\sqrt{2} = 17.0$ (3sf) **[1mark]**
$17 - 10 = 7$ **[1mark]**
Therefore, $x = 7 \div 2 = 3.5$ cm (1dp) **[1mark]**

**4** ABCD is a square of side 1 cm

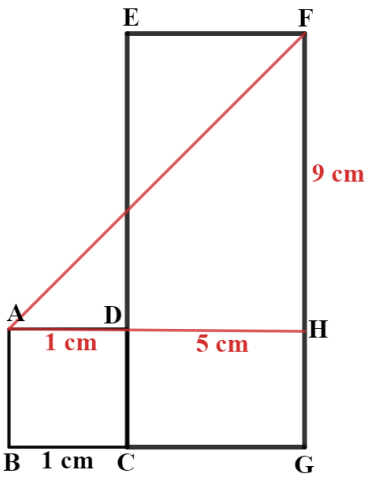



CEFG is a rectangle of area 50 $cm^2$
Calculate the direct distance between A and F
Give your answer as a simplified surd

**Solution**

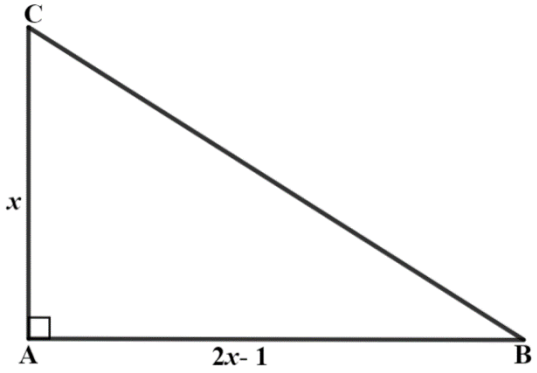
The width of the rectangle will be $= 50 \div 10 = 5$ cm **[1mark]**

Notice that since the side length of the square is 1 cm.
Tt means that $HF = 10 - 1 = 9$ cm
Also, AH will be $5 + 1 = 6$ cm
Apply Pythagoras theorem to triangle AHF




$AF^2 = 9^2 + 6^2$ **[1mark]**
$= 81 + 36$
$= 117$
Therefore, $AF = \sqrt{117} = 3\sqrt{13}$ **[2marks]**
{Note that $\sqrt{117} = \sqrt{9} \times \sqrt{13} = 3\sqrt{13}$}

**5** ABC is a right-angled triangle




$AC = x$

$AB = 2x - 1$

Given that the area of the triangle is 3 cm$^2$,
calculate the exact perimeter of the triangle.

**Solution**

Area of a triangle $= \frac{1}{2}\text{b} \times \text{h}$

Therefore, $\frac{1}{2}\text{b} \times \text{h} = 3$

$\text{b} \times \text{h} = 6$ **[1mark]**

Therefore, $x(2x - 1) = 6$ **[1mark]**

$2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$ **[1mark]**

Therefore, $x = -\frac{3}{2}$ or $x = 2$ **[1mark]**

Therefore, the true value of $x$ is 2.

The height of the triangle is 2 and the base is 3

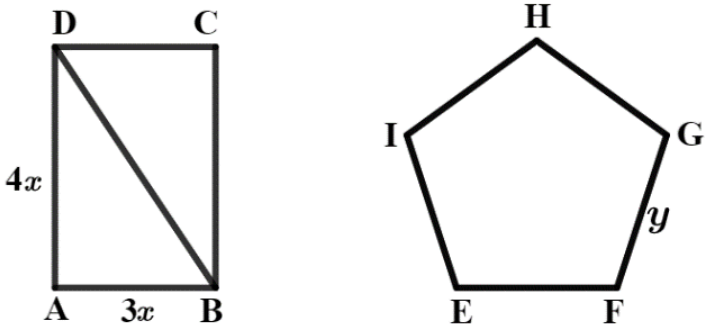
$l^2 = 2^2 + 3^2$ where $l$ is the longer side of the triangle.

$l = \sqrt{4 + 9} = \sqrt{13}$ **[1mark]**

Hence, the exact perimeter of the triangle is

$2 + 3 + \sqrt{13} = 5 + \sqrt{13}$ **[1mark]**

**6** ABCD is a rectangle
EFGHI is a regular pentagon




$AB = 3x$
$AD = 4x$
$BD = 10\sqrt{5}$ cm
$FG = y$
The two shapes have the same perimeter
Find the values of $x$ and $y$
Give your answers to 3 significant figures

**Solution**

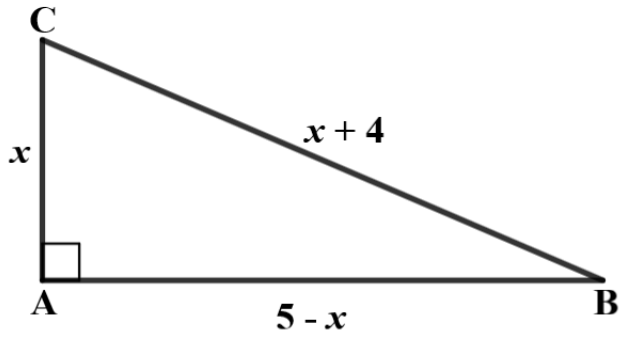
The perimeter of the rectangle in terms of $x$ will be
$2 \times 4x + 2 \times 3x = 14x$ **[1mark]**

$(4x)^2 + (3x)^2 = \left(10\sqrt{5}\right)^2$
$16x^2 + 9x^2 = 500$
$25x^2 = 500$
$x^2 = 20$
$x = 4.47$ (3sf) **[2marks]**

Therefore, the perimeter of the rectangle is
$14 \times 4.47 = 62.58$ **[1mark]**

The perimeter of the pentagon $= 5y$
Therefore, $5y = 62.58$ **[1 mark]**
Hence, $y = 12.5$ (3sf) **[1 mark]**

**7** ABC is a right-angled triangle




$AB = 5 - x$
$AC = x$
$BC = x + 4$
Calculate the value of $x$
Give your answer to 3 decimal places

**Solution**

Apply Pythagoras theorem
$x^2 + (5 - x)^2 = (x + 4)^2$ **[1mark]**
$x^2 + 25 - 10x + x^2 = x^2 + 8x + 16$
$2x^2 - 10x + 25 = x^2 + 8x + 16$
$x^2 - 18x + 9 = 0$ **[1mark]**
Now use the quadratic formula to solve
$a = 1; b = -18$ and $c = 9$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{--18 \pm \sqrt{(-18)^2 - 4 \times 1 \times 9}}{2 \times 1}$
$x = 17.5$ or $0.515$ (3dp) **[2marks]**
Therefore, $x = 0.515$
The true value of $x$ is 0.515 since 17.5 will make the base of the triangle negative. **[1mark]**