FluidMaths

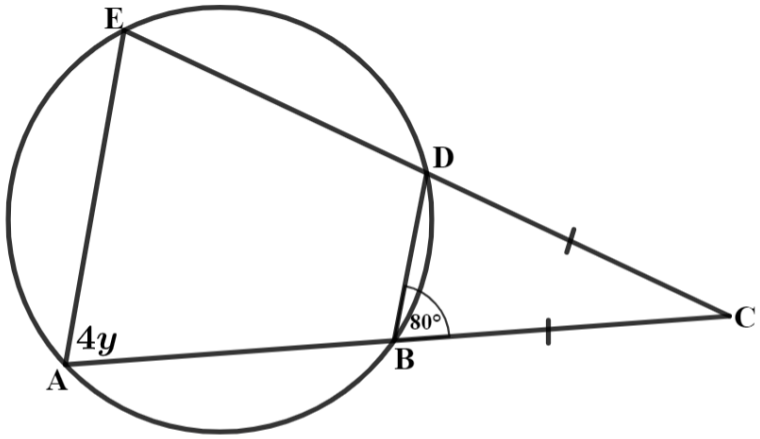
GCSE Mathematics (Grade 9-1)

Problem Solving
Circle Theorems
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 A, B, D and E are points on the circumference of a circle

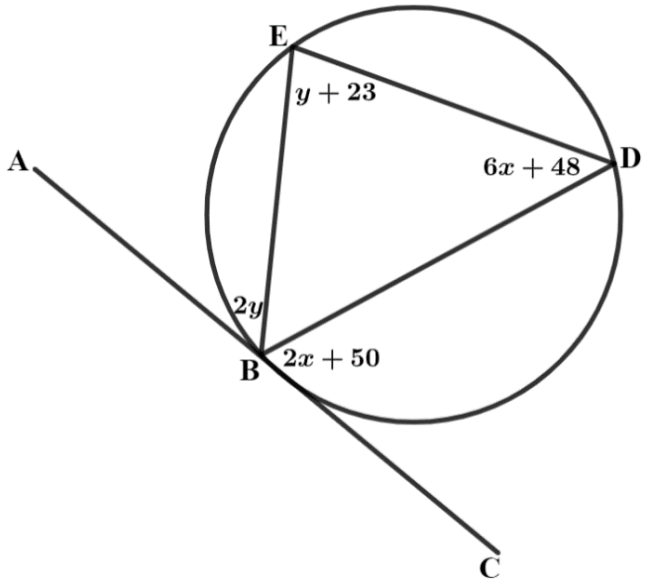


$BC = CD$
AC and CE are straight lines
Angle $CBD = 80°$
Angle $BAE = 4y$
Calculate the value of y?

Solution

Angle $BDC = 80°$
{Angles in an isosceles triangle}
Angle $EDB = 180 - 80 = 100$ **[1mark]**
$4y = 100$ **[1mark]**
{Opposite angles in a cyclic quadrilateral add up to 180°}
Therefore, $y = 25$ **[1mark]**

2 In the diagram below, ABC is a tangent to the circle



Angle ABE $= 2y$
Angle CBD $= 2x + 50$
Angle BDE $= 6x + 48$
Angle BED $= y + 23$
Calculate the values of x and y

Solution

The alternate segment theorem

$2x + 50 = y + 23$

$2x - y = -27$ ----------equation 1 **[1mark]**

$2y = 6x + 48$

$y = 3x + 24$----------equation 2 **[1mark]**

Substitute equation 2 into equation 1

$2x - (3x + 24) = -27$ **[1mark]**

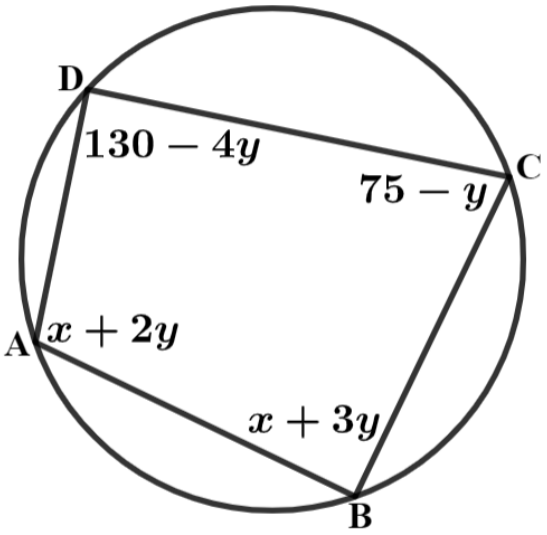
$2x - 3x - 24 = -27$

$-x = -3$

Therefore, $x = 3$ **[1mark]**

Hence, $y = 3 \times 3 + 24 = 33$ **[1mark]**

3 A, B, C and D are points on the circumference of the circle

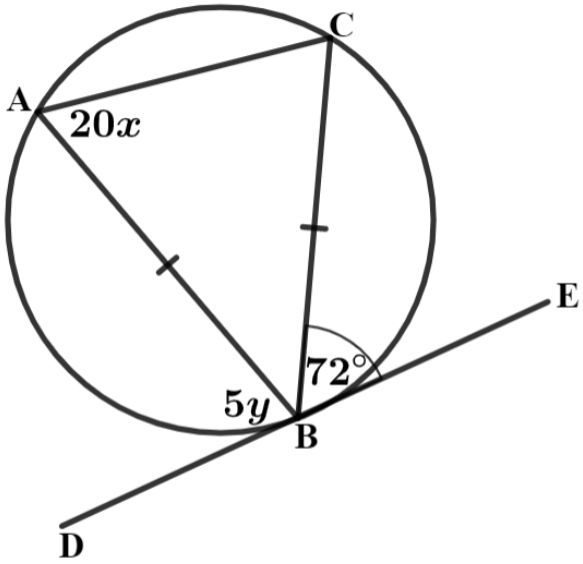


Angle BAD $= x + 2y$
Angle BCD $= 75 - y$
Angle ABC $= x + 3y$
Angle CDA $= 130 - 4y$
Calculate the ratio of $x : y$ in its simplest form

Solution

Opposite angles in a cyclic quadrilateral add up 180°
$x + 2y + 75 - y = 180$
$x + y = 105$ ----------equation 1 **[1mark]**
$x + 3y + 130 - 4y = 180$
$x - y = 50$ ---------- equation 2 **[1mark]**
Add equations 1 and 2
$x + y = 105$
$\underline{x - y = 50}$
$2x = 155$
$x = 77.5$ **[2marks]**
Therefore, $77.5 + y = 105$
$y = 27.5$ **[1mark]**
Hence, $x : y = 77.5 : 27.5 = 31 : 11$ **[1mark]**

4 ABC is a triangle inscribed inside a circle

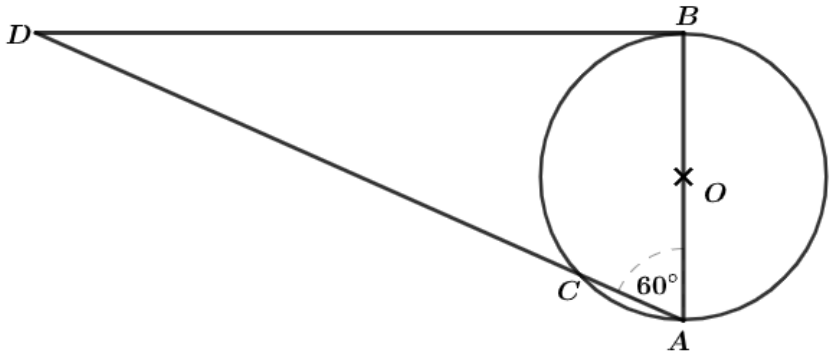


AB = BC
Angle CBE = 72°
Angle ABD = $5y$
Angle BAC = $20x$
DE is a tangent to the circle at B
Find the ratio of $x : y$ in its simplest form

Solution

The alternate segment theorem
$20x = 72$
$x = 3.6$ **[1mark]**
Triangle ABC is an isosceles triangle
AB = BC therefore, Angle BAC = Angle BCA
Therefore, $5y = 72$
$y = 14.4$ **[1mark]**
$x : y = 3.6 : 14.4 = 1 : 4$ **[2marks]**

5 The diagram below shows a circle with center O.



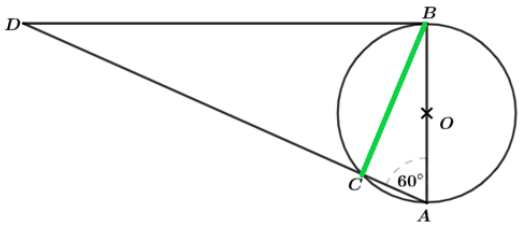
The circumference of the circle is 10π cm
BD is a tangent to the circle and AC is a chord
Calculate the length of CD to 1 decimal place.

Solution

Angle ABD is 90° {Angle between a circle and a radius}
Circumference $= \pi d$
Therefore $10\pi = \pi d$
$d = 10$
Therefore, AB = 10 cm **[1mark]**
Consider triangle ABD

$$\cos 60 = \frac{10}{AD}$$

$$AD = \frac{10}{\cos 60} = 20 \text{ cm} \quad \textbf{[1mark]}$$

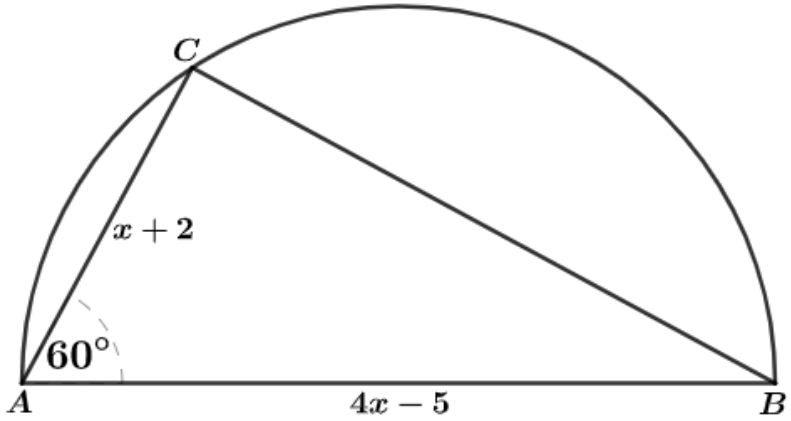


Extend a line between points C and B.
Notice that triangle ABC is right-angled at C {Angle in a semi-circle is 90°}
Consider triangle ABC

$$\cos 60 = \frac{AC}{10}$$

$AC = 10 \times \cos 60 = 5$ **[1mark]**
Hence, CD = 20 − 5 = 15 cm **[1mark]**

6 Triangle ABC is drawn inside a semi-circle of diameter AB



AB $= 4x - 5$
AC $= x + 2$
Calculate the perimeter of the semi-circle
Give your answer to 3 significant figures.

Solution

{Angle is a semi-circle is 90°}
Therefore, ABC is a right-angled triangle

$\cos 60 = \frac{x+2}{4x-5}$

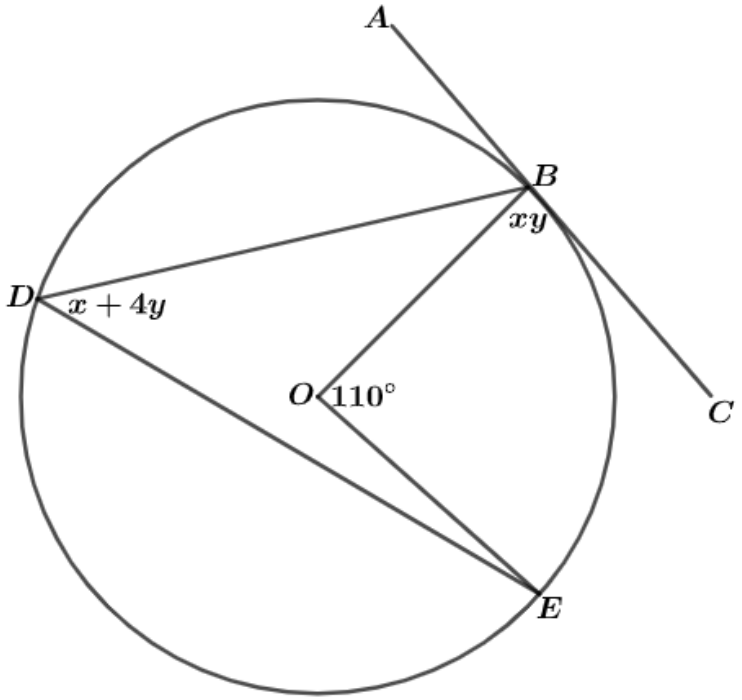
$\frac{1}{2} = \frac{x+2}{4x-5}$ **[1mark]**

$4x - 5 = 2x + 4$
$2x = 9$
$x = \frac{9}{2} = 4.5$ **[1mark]**

The diameter AB $= 4 \times 4.5 - 5 = 13$ **[1mark]**

{circumference of a semi-circle $= \pi d \div 2$}
Therefore, we have $13\pi \div 2 = 20.4$ (3sf) **[1mark]**
Hence, the Perimeter of the semi-circle is
$20.4 + 13 = 33.4$ (3sf) **[1mark]**

7 AC is a tangent to the circle with centre O



Angle BDE $= x + 4y$
Angle OBC $= xy$
Angle BOE $= 110°$
Calculate the values of x and y to the nearest whole number

Solution

{Angle between a radius and a tangent}
Therefore, $xy = 90$----------equation 1 **[1mark]**
{Angle at the circumference is half the angle at the centre}
$x + 4y = 55$
$x = 55 - 4y$ ----------equation 2 **[1mark]**
Now substitute equation 2 into equation 1
$y(55 - 4y) = 90$
$55y - 4y^2 = 90$
$4y^2 - 55y + 90 = 0$ **[1mark]**
Solve using the quadratic formula:
$a = 4; b = -55$ and $c = 90$

	$y = \frac{--55 \pm \sqrt{(-55)^2 - 4 \times 4 \times 90}}{2 \times 4}$ $y = 12$ or $y = 2$ (To nearest whole number) **[1mark]** $x = 55 - 4 \times 12 = 7$ or $x = 55 - 4 \times 2 = 47$ **[1mark]**