FluidMaths

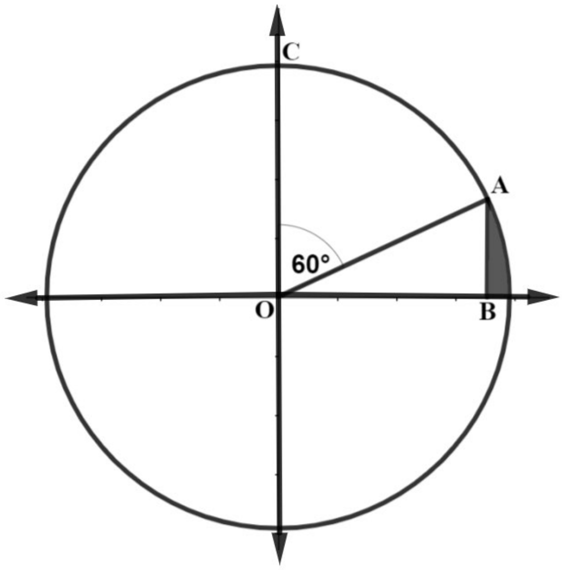
GCSE Mathematics (Grade 9-1)

Problem Solving
Area of a Circle Set 3
Sectors and Segments
Solutions

The marks shown are for guidance purposes only

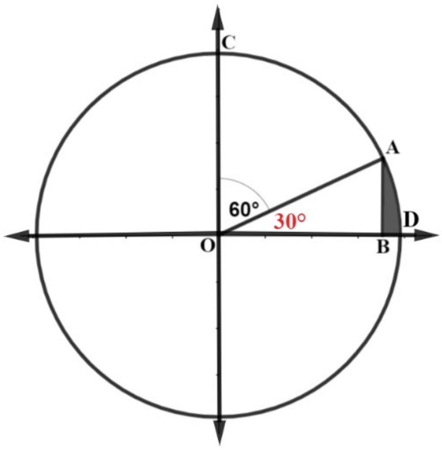
The questions are repeated here for your convenience

1	The diagram below shows a circle of radius of 3 cm



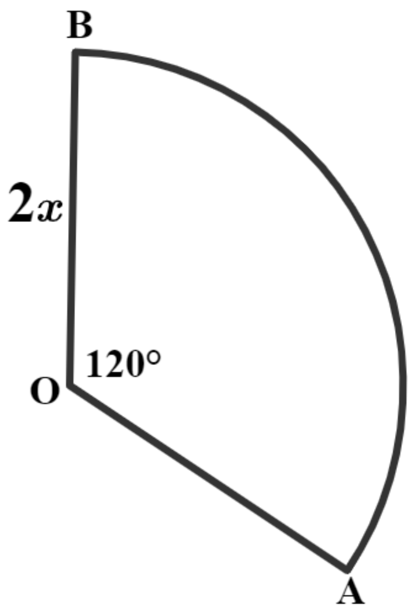
Angle AOC $= 60°$
Angle ABO $= 90°$
Calculate the shaded area to 1 decimal place

Solution



OA $= 3$ cm
Angle AOB $= 30°$
Calculate OB using SOHCAHTOA
$\cos 30 = \frac{\text{OB}}{3}$
OB $= 3 \times \cos 30 = 2.60$ cm**[1mark]**
Calculate the area of triangle OBA
Use the sine rule {Area of a triangle $= \frac{1}{2}ab \times \sin \text{C}$}
The area of triangle AOB $= \frac{1}{2} \times 3 \times 2.6 \times \sin 30 = 1.95\ \text{cm}^2$
[1mark]
Area of the sector ODA $= \frac{30}{360} \times \pi \times 3^2 = 2.36\ \text{cm}^2$**[1mark]**
Therefore, the shaded are $= 2.36 - 1.95 = 0.41\ \text{cm}^2$(2dp)
[1mark]

2 ABO is a sector of radius $2x$



Angle AOB $= 120°$

Given that the perimeter of the sector is 25 cm, calculate the value of x

Give your answer to 1 decimal place.

Solution

Perimeter of a sector = arc length + 2 (radii)

length of arc AB $= \frac{120}{360} \times 2 \times 2x \times \pi = \frac{4}{3}\pi x$ **[1mark]**

The perimeter of the sector is $= \frac{4}{3}\pi x + 2x + 2x$ **[1mark]**

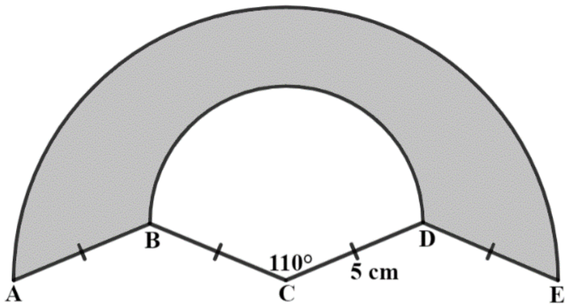
Therefore, $\frac{4}{3}\pi x + 4x = 25$ **[1mark]**

$4\pi x + 12x = 75$ **[1mark]**

$x(4\pi + 12) = 75$

$x = \frac{75}{4\pi+12} = 3.1$ (1dp) **[1mark]**

3 The diagram below shows a semi-circle and a sector



$AB = BC = CD = DE = 5\text{cm}$

Angle ABC $=$ Angle BCD $=$ Angle CDE $= 110°$

AE is the diameter of the semi-circle.

C is the midpoint of AE

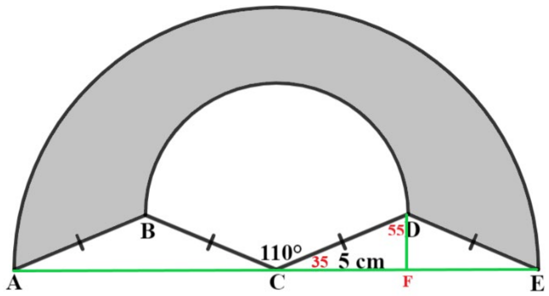
Calculate the shaded area

Give your answer to 3 significant figures.

Solution

$\left\{\text{The area of a sector} = \frac{\theta}{360} \times \pi r^2\right\}$

{Refer to the illustration in the RHS}



The area of the sector$=$ $\frac{110}{360} \times \pi \times 5^2 = 24.0 \text{ cm}^2$ **[1mark]**

The area of triangle ABC$= \frac{1}{2} \times 5^2 \times \sin 110 = 11.7$ **[1mark]**

Triangle ABC is congruent to triangle CDE

Therefore, the area of the 2 triangles is $2 \times 11.7 = 23.4$ **[1mark]**

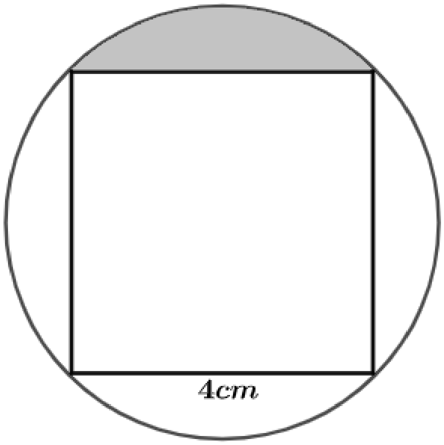
From triangle CEF $CF = 5 \times \cos 35 = 4.1$ cm

The radius of the semi-circle $= 2 \times 4.1 = 8.2$ cm **[1mark]**

The area of the semi-circle $= \frac{\pi \times 8.2^2}{2} = 106\text{cm}^2$ **[1mark]**

The area of the shaded region is $106 - 24 - 23.4 =$ 58.6 cm^2 **[1mark]**

4 The diagram shows a square of side 4 cm and a circle.
The vertices of the square lie on the circumference of the circle



Show that the exact area of the shaded segment is $2(\pi - 2)$

Solution

Use Pythagoras theorem to find the diagonal of the square.
Let the diagonal of the square be d

Then $4^2 + 4^2 = d^2$ **[1mark]**

$d^2 = 32$

$d = \sqrt{32} = 4\sqrt{2}$ $\{\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}\}$ **[1mark]**

The diameter of the circle is $4\sqrt{2}$ so the radius will be $2\sqrt{2}$

The area of the circle is $\{A = \pi r^2\}$

Therefore, $A = \pi\left(2\sqrt{2}\right)^2 = 8\pi$ **[1mark]**

The area of the square is $4 \times 4 = 16$

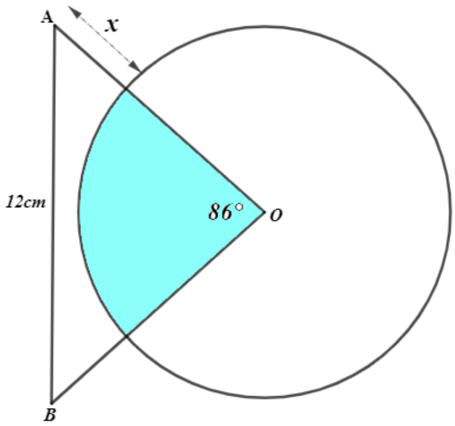
The area of the four segments will be $8\pi - 16$ **[1mark]**

The area of the shaded region is $= \frac{1}{4}(8\pi - 16) = 2\pi - 4$

{factorise to obtain required form}

That is, $2(\pi - 2)$ **[1mark]**

5 The diagram below shows a circle with centre O.
ABO is an isosceles triangle.



Where AO = BO and angle AOB is 86°.
The area of the shaded sector is 28.85 cm²
Calculate the value of x
Give your answer to 1 significant figure

Solution

$\left\{\text{The area of a sector} = \frac{\theta}{360} \times \pi r^2\right\}$

Therefore, $28.85 = \frac{86}{360} \times \pi \times r^2$ **[1mark]**

$\frac{28.85 \times 360}{86 \times \pi} = r^2$

$r = \sqrt{38.4} = 6.2$ cm **[1mark]**

From triangle ABO, Let AO $= a$

Use the cosine rule:$\{ a^2 = b^2 + c^2 - 2bc \times \cos A\}$

Therefore, $12^2 = a^2 + a^2 - 2 \times a \times a \times \cos 86$

$144 = 2a^2 - 0.14a^2$

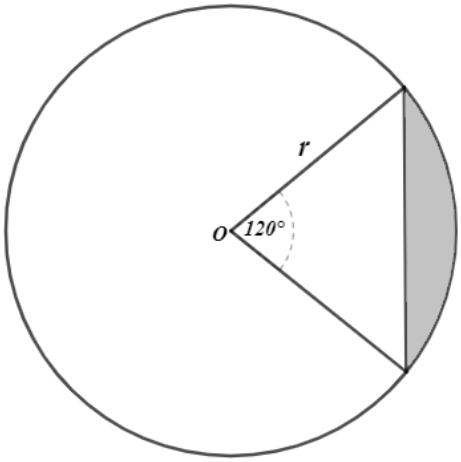
$144 = 1.86a^2$ **[1mark]**

$a = \sqrt{\frac{144}{1.86}} = 8.8$ cm **[1mark]**

Therefore, $x = 8.8 - 6.2 = 2.5$ **[1mark]**

Hence, $x = 3$ cm(1sf) **[1mark]**

6 The diagram below shows a circle with centre O



The radius of the circle is r.
The area of the shaded segment is 12.5 cm^2
Calculate the value of r to 1 decimal place.

Solution

$\left\{\text{The area of a sector} = \frac{\theta}{360} \times \pi r^2\right\}$ and $\left\{\text{The area of a triangle} = \frac{1}{2}\text{abSin C}\right\}$

The area of the segment is
The area of the sector – the area of the triangle

That is, the area of a segment $= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2}ab \times \sin C$

[1mark]

So, $12.5 = \left(\frac{120}{360} \times \pi r^2\right) - \left(\frac{1}{2} \times r^2 \times \sin 120\right)$ **[1mark]**

$12.5 = \left(\frac{1}{3} \times \pi r^2\right) - \left(\frac{1}{2} \times r^2 \times \frac{\sqrt{3}}{2}\right)$ **[1mark]**

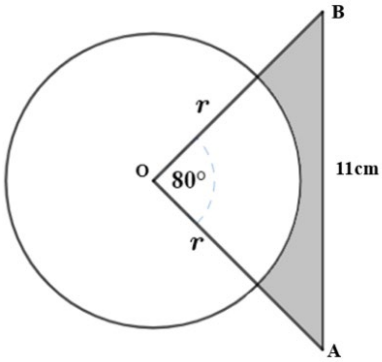
$12.5 = \frac{1}{3}\pi r^2 - r^2 \frac{\sqrt{3}}{4}$ **[1mark]** {Factorise r^2}

$12.5 = r^2\left(\frac{1}{3}\pi - \frac{\sqrt{3}}{4}\right)$

$12.5 = 0.614r^2$ **[1mark]**

$r = \sqrt{\frac{12.5}{0.614}} = 4.5$ cm (1dp) **[1mark]**

7 The diagram shows a circle with centre O and radius *r*.



AO = BO

AB = 11 cm

The shaded area is 6 cm^2

Calculate the radius of the circle to 1 decimal place.

Solution

Apply the cosine rule: $\{a^2 = b^2 + c^2 - 2bc \times \cos A\}$

Let $\text{AO} = \text{BO} = x$ then,

$11^2 = x^2 + x^2 - 2 \times x \times x \times \cos 80$

$121 = 2x^2 - 0.347x^2$

$121 = 1.65x^2$ **[1mark]**

$x = \sqrt{\frac{121}{1.65}} = 8.56 \text{ cm}$ **[1mark]**

Therefore, the area of the triangle is

$\{\text{The area of a triangle} = \frac{1}{2}ab \times \sin \text{C}\}$

$\text{A} = \frac{1}{2} \times 8.56 \times 8.56 \times \sin 80 = 36.1$ **[1mark]**

The area of the sector will be $= 36.1 - 6 = 30.1$ **[1mark]**

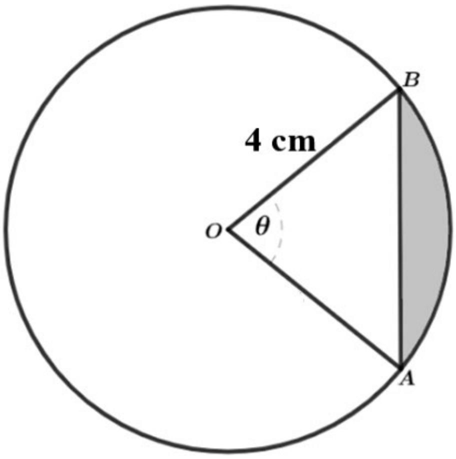
$\left\{\text{The area of a sector } = \frac{\theta}{360} \times \pi r^2\right\}$

Therefore, $30.1 = \frac{80}{360} \times \pi \times r^2$ {rearrange to find *r*}

$r^2 = \frac{360 \times 30.1}{80\pi} = 43.1$ **[1mark]**

$r = \sqrt{43.1} = 6.6 \text{ cm (1dp)}$ **[1mark]**

8 The diagram shows a circle with centre O.



The radius of the circle is 4 cm

Given that AB = 2.1 cm, what is the size of the shaded area?

Give your answer to 2 decimal places.

Solution

Use cosine rule:$\{a^2 = b^2 + c^2 - 2bc \times \cos A\}$

to find the angle in the sector. $\left\{\cos A = \frac{b^2+c^2-a^2}{2bc}\right\}$

Therefore, $\cos\theta = \frac{4^2+4^2-2.1^2}{2\times4\times4} = 0.862$ **[1mark]**

Therefore, $\theta = \cos^{-1}(0.862) = 30.4°$ **[1mark]**

The area of the triangle$= \frac{1}{2} \times 4 \times 4 \times \sin 30.4 = 4.05$ **[1mark]**

Area of the sector $= \frac{30.4}{360} \times \pi \times 4^2 = 4.24$ **[1mark]**

So, the area of the segment is $4.24 - 4.05 = 0.19$(3sf) **[1mark]**