FluidMaths

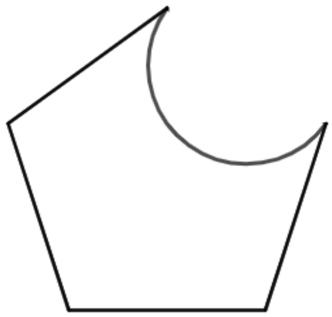
GCSE Mathematics (Grade 9-1)

Problem Solving
Area of a Circle Set 2
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 A semi-circle is removed from one side of a regular pentagon as shown below



If the area of the semi-circular is 25 cm², find the perimeter of the shape
Give your answer to 3 significant figures.

Solution

The diameter of the semi-circle is also the side of the pentagon.

$$\left\{\text{Area of a semi} - \text{circle } = \frac{1}{2} \times \pi \times r^2\right\}$$

Therefore, $\frac{1}{2} \times \pi \times r^2 = 25$

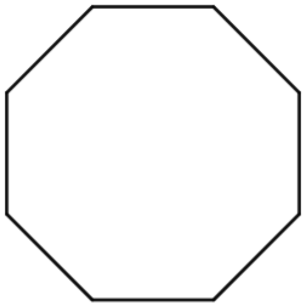
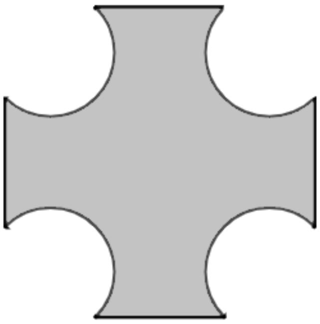
$$r^2 = \frac{25 \times 2}{\pi}$$

$r = \sqrt{\frac{50}{\pi}} = 3.99\ (3\text{sf})$ **[1mark]**

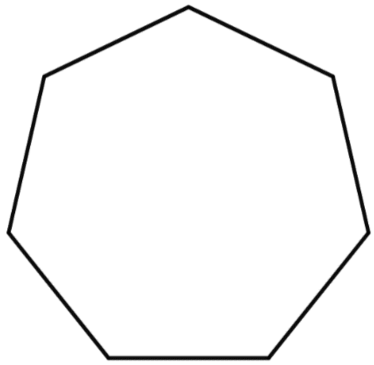
Therefore, the diameter will be $2 \times 3.99 = 7.98$

Circumference of the semi-circle will be
$(7.98 \times \pi) \div 2 = 12.5(3\text{sf})$ **[2marks]**

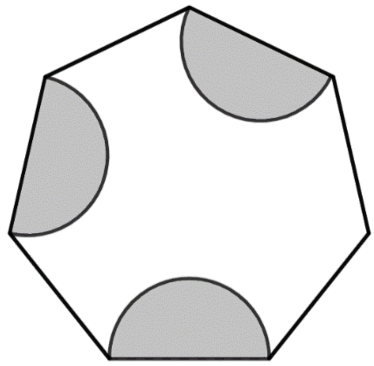
Hence the perimeter of the shape will be
$4 \times 7.98 + 12.5 = 44.4\ \text{cm}(3\text{sf})$ **[1mark]**

2	A regular octagon is shown below  The area of the octagon is 150 cm^2 The perimeter of the octagon is 48 cm Four identical semi-circles have been removed from the octagon to form the new shape below  Calculate the area of the new shape. Give your answer in terms of π **<u>Solution</u>** Each side of the octagon = $48 \div 8 = 6$ cm Therefore, the diameter of each semi-circle will be 6 cm. So, the radius is 3 cm. **[1mark]** The area of each semi-circle = $\pi \times 3^2 \div 2 = 4.5\pi$ **[1mark]** The total area of the 4 semi-circles = $4 \times 4.5\pi = 18\pi$ **[1mark]** Hence the area of the new shape = 150 – 18π **[1mark]**

3 Here is a regular heptagon



The perimeter of the heptagon is 70 cm
Three semi-circles are drawn inside the heptagon as shown

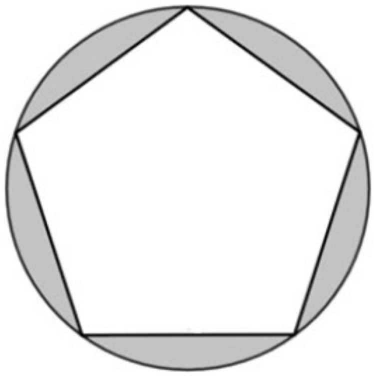


30% of the heptagon is shaded
What is the area of the heptagon?
Give your answer in terms of pi.

Solution

The diameter of each semi-circle will be $70 \div 7 = 10$ cm
Therefore, the radius will be 5 cm **[1mark]**
The area of one semi-circle $= 5^2 \times \pi \div 2 = 12.5\pi$ **[1mark]**
Therefore, the total shaded area will be $3 \times 12.5\pi = 37.5\pi$ **[1mark]**
Let the area of the heptagon be A, then 30% of $A = 37.5\pi$
Therefore, $A = 37.5\pi \div 0.3 = 125\pi$ **[2marks]**
Hence, the area of the heptagon is 125π cm^2

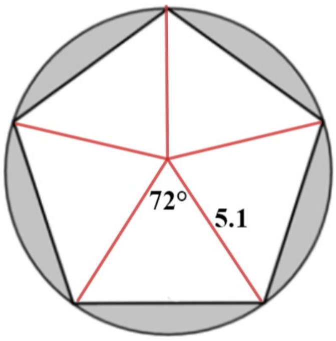
4 A regular pentagon is drawn inside a circle as shown



The pentagon and the circle have the same centre
The radius of the circle is 5.1 cm
Calculate the size of the shaded area
Give your answer to 3 significant figures

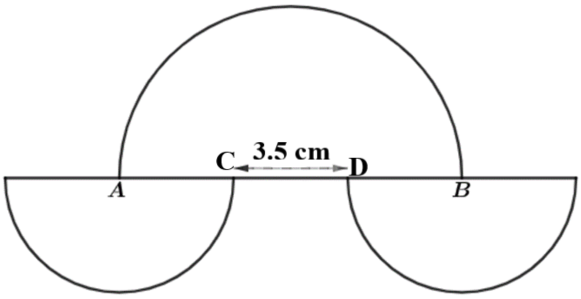
Solution

Split the pentagon into 5 identical isosceles triangles.
The angle at the center is found as $360 \div 5 = 72$ **[1mark]**



Calculate the area of each triangle using
$\left\{\textbf{The area of a triangle} = \frac{1}{2}\textbf{ab} \times \textbf{sin C}\right\}$
$A = \frac{1}{2} \times 5.1 \times 5.1 \times \sin 72 = 12.4\ (3sf)$ **[1mark]**
Therefore, the area of the pentagon will be
$5 \times 12.4 = 61.8\ cm^2$ **[1mark]**
The area of the circle $= \pi \times 5.1^2 = 81.7(3sf)$ **[1mark]**
Hence, the shaded area $= 81.7 - 61.8 = 19.9\ cm^2$ **[1mark]**

5 Two identical semi-circles have their centers at points A and B as shown in the diagram below.



CD = 3.5 cm

AB is the diameter of the larger semi-circle

The area of the larger semi-circle is 32π cm^2.

Calculate the perimeter of the shape in terms of π.

Solution

Area of a semi − circle $= \frac{1}{2} \times \pi r^2$

Therefore, $32\pi = \frac{1}{2} \times \pi r^2$ **[1mark]**

$64\pi = \pi r^2$

$r^2 = 64$

$r = 8$

Therefore, the diameter is AB is16 cm **[1mark]**

Therefore, the diameter of the smaller semi-circle is

16 – 3.5 =12.5 **[1mark]**

Because the 2 semi-circles are identical, we only need to calculate the circumference of a circle

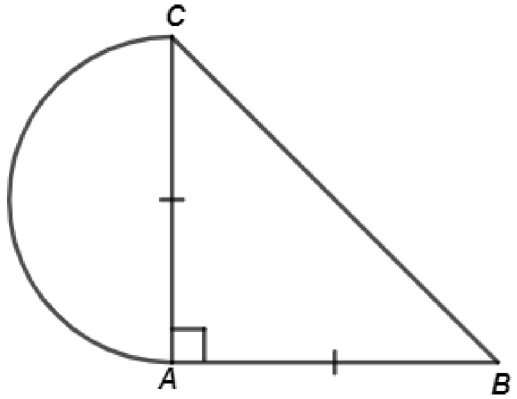
That is, Circumference $= 12.5\pi$ **[1mark]**

The circumference of the larger semi-circle $= 16\pi \div 2 = 8\pi$ **[1mark]**

Hence, the perimeter of the shape is

$8\pi + 12.5\pi + 12.5 + 3.5 = 20.5\pi + 16$ **[1mark]**

6 ABC is an isosceles triangle



AB = AC

A semi-circle is drawn to the side AC

The area of the semi-circle is 32π cm^2

Show that the area of triangle ABC is 128 cm^2

Solution

$\left\{\text{Area of a semi} - \text{circle} = \frac{1}{2}\pi r^2\right\}$

Therefore, $32\pi = \frac{\pi r^2}{2}$ **[1mark]**

$64\pi = \pi r^2$

$64 = r^2$

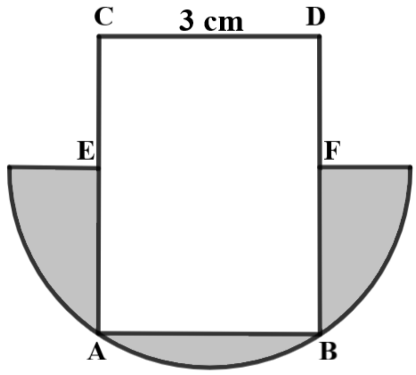
Therefore, $r = 8$ **[1mark]**

Therefore, $\text{AC} = 2 \times 8 = 16$ cm **[1mark]**

Hence, the area of triangle ABC will be

$\frac{1}{2} \times 16 \times 16 = 128\ \text{cm}^2$ **[1mark]**

7 Rectangle ABCD is drawn inside a semi-circle as shown



The circumference of the semi-circle is 4π cm

CD = 3 cm

BC = 5 cm

$\frac{3}{4}$ of the rectangle is inside the semi-circle

Calculate the exact size of the shaded area.

Solution

Use Pythagoras theorem to find the height of the rectangle

$AC^2 = 5^2 - 3^2$

$AC = \sqrt{25 - 9} = 4$ **[1mark]**

Therefore, $AE = \frac{3}{4} \times 4 = 3$ cm **[1mark]**

So, the area of the rectangle that is inside the semi-circle is $3 \times 3 = 9$ cm² **[1mark]**

The circumference of a semi-circle is $\frac{\pi d}{2}$

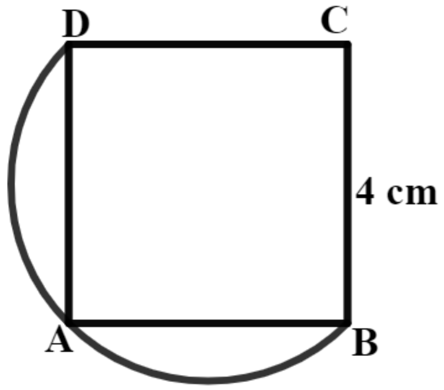
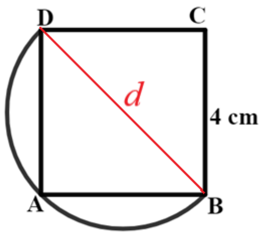
Therefore, $\frac{\pi d}{2} = 4\pi$

So, $d = 8$ cm **[1mark]**

The radius of the semi-circle will be 4 cm

The area of the semi-circle $= \frac{1}{2}\pi \times 4^2 = \frac{1}{2}\pi \times 16 = 8\pi$ **[1mark]**

Therefore, the shaded area $= (8\pi - 9)$ cm² **[1mark]**

8	ABCD is a square of side 4 cm A semi-circle is drawn to the square such that BD is the diameter of the semi-circle  Show that the perimeter of the shape can be written as $2(4 + \pi\sqrt{2})$ **Solution** The diagonal of the square is the diameter of the semi-circle.  Let d be the diagonal of the square then $d^2 = 4^2 + 4^2$ $d^2 = 32$ $d = \sqrt{32} = 4\sqrt{2}$ **[2marks]** Circumference of the semi-circle $C = \pi \times 4\sqrt{2} \div 2 = 2\pi\sqrt{2}$ **[1mark]** Hence, the perimeter of the shape will be $2\pi\sqrt{2} + 8 = 2(4 + \pi\sqrt{2})$ **[2marks]**