FluidMaths

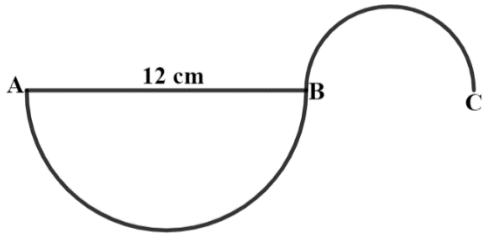
GCSE Mathematics (Grade 9-1)

Problem Solving
Area of a Circle Set 1
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 The diagram below shows two semi-circles
AB is the diameter of the larger semi-circle
A, B and C are in a straight line



AB = 12 cm
If AB: BC = 2: 1, what is the exact perimeter of the shape?

Solution

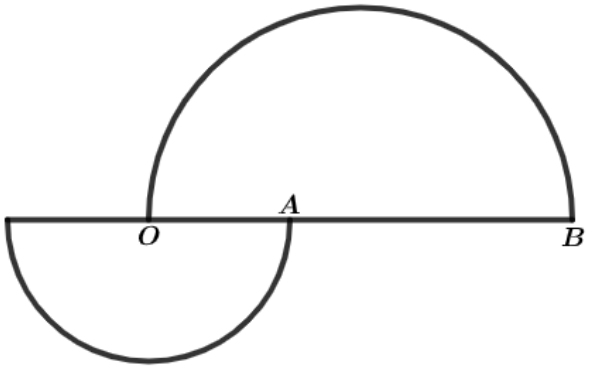
The circumference of the larger semi-circle is
$= 12\pi \div 2 = 6\pi$ **[1mark]**

The diameter of the smaller semi-circle is $12 \div 2 = 6$
[1mark]

Therefore, the circumference of the smaller semi-circle is
$= 6\pi \div 2 = 3\pi$ **[1mark]**

Hence, the perimeter of the shape will be
$6\pi + 3\pi + 12 = 9\pi + 12$ **[1mark]**

2 Two semi-circles are shown below



The circumference of the larger semi-circle is 18π cm
If OA: OB = 1: 4, calculate the perimeter of the shape
Give your answer in terms of π

Solution

Circumference = πd

Therefore, $18\pi = \frac{1}{2} \times \pi d$ {Since it's a semi-circle}

$d = 36$

So, diameter OB = 36 cm **[1mark]**

OA is the radius of the smaller semi-circle

OA: OB = 1: 4

Therefore, $\frac{\text{OA}}{\text{OB}} = \frac{1}{4}$

$\text{OA} = \frac{1}{4} \times 36 = 9$ **[1mark]**

So, the diameter of the smaller semi-circle will be

$2 \times 9 = 18$ **[1mark]**

Circumference of the smaller semi-circle

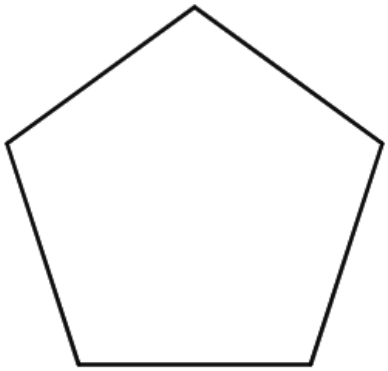
$= \frac{1}{2} \times \pi d$

$= \frac{1}{2} \times \pi \times 18 = 9\pi$

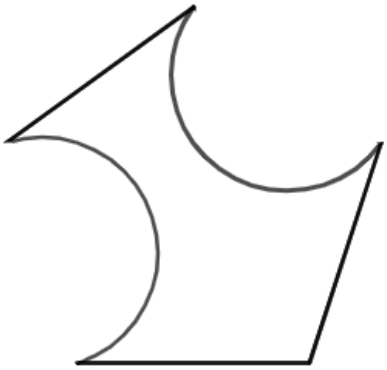
The perimeter of the shape $= 9\pi + 18\pi + 36 = 27\pi + 36$

[1mark]

3 Here is a regular pentagon



The perimeter of the pentagon is 24 cm
Two semi-circles are removed from the pentagon to form the new shape below

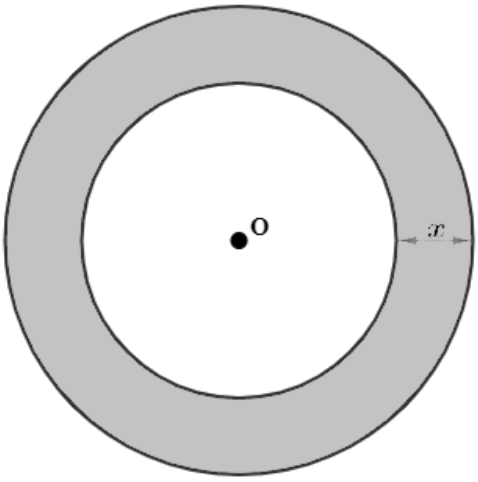


Calculate the perimeter of the new shape
Give your answer to 1 decimal place.

Solution

Side length of the pentagon is $24 \div 5 = 4.8$ cm **[1mark]**
Therefore, the diameter of each semi-circle is 4.8 cm
Since we have two semi-circles, we only need to calculate the circumference of a whole circle.
Circumference $= \pi d$
So, $C = 4.8\pi$ **[1mark]**
Therefore, the perimeter of the shape will be
$4.8\pi + 4.8 + 4.8 + 4.8 = 14.4 + 4.8\pi = 29.5$ cm(1dp)
[1mark]

4 Here are two circles



The two circles have the same centre at O
The distance between their circumferences is x
The area of the smaller circle is 48 cm^2
The area of the shaded region is 18 cm^2
Calculate the value of x to 2 decimal places.

Solution

The area of the shaded region is
The area of the larger circle – the area of the smaller circle
Therefore, the area of larger circle $- 48 = 18$
So, the area of the larger circle $= 66$ **[1mark]**
{Area of a circle $= \pi r^2$}
For the larger circle
$66 = \pi r^2$

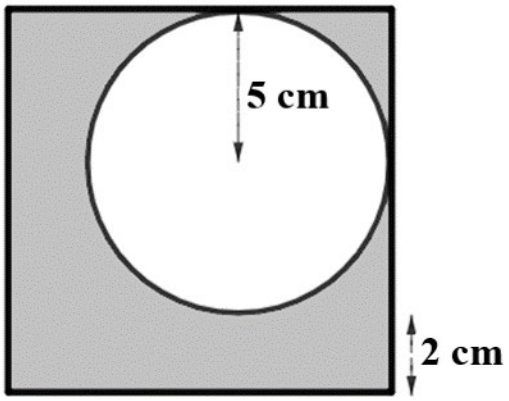
$r = \sqrt{\frac{66}{\pi}} = 4.58$ (3sf) **[1mark]**

For the smaller circle
$48 = \pi r^2$

$r = \sqrt{\frac{48}{\pi}} = 3.91$ (3sf) **[1mark]**

Hence, $x = 4.58 - 3.91 = 0.67$ cm(2dp) **[1mark]**

5 A circle of radius 5 cm is drawn inside a square so that it touches two sides of the square and is exactly 2 cm from the other two sides of the square as shown below.



What percentage of the area of the square is shaded?
Give your answer to 1 decimal place.

Solution

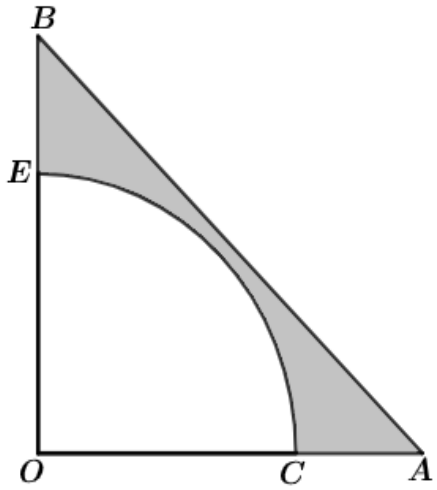
The area of a circle $\{A = \pi r^2\}$
Therefore, $A = \pi \times 5^2 = 25\pi$ **[1mark]**

Side length of the square $= 5 + 5 + 2 = 12$ cm **[1mark]**
Area of the square $= 12 \times 12 = 144$ **[1mark]**

The shaded area $= 144 - 25\pi = 65.5$ **[1mark]**

% of shaded area $= \frac{65.5}{144} \times 100 = 45.5\%$ **[1mark]**

6 OAB is a right-angled triangle



The arc CE is 5π cm

The ratio of OC: OA = 2 : 3

Calculate the size of the shaded region to 2 decimal places

Solution

OAB is a right-angled triangle

Therefore, CE is $\frac{1}{4}$ of the circumference of a circle.

Therefore, $5\pi = \frac{1}{4} \times 2 \times \pi \times r$ **[1mark]**

$r = 10$ **[1mark]**

Therefore, OC = 10 cm

OC : OA = 2 : 3 therefore, $\frac{OA}{OC} = \frac{3}{2}$

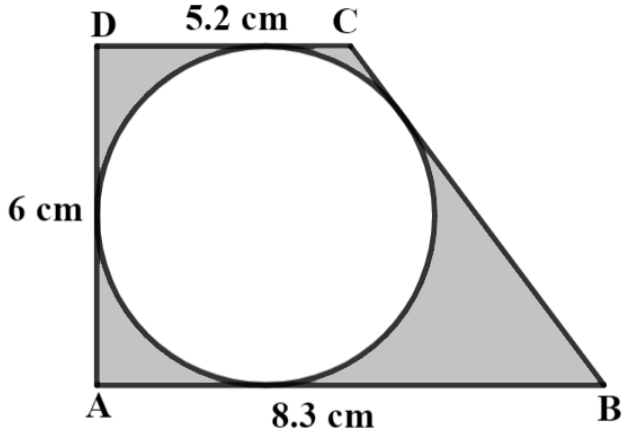
So, $OA = \frac{3}{2} \times 10 = 15$ cm **[1mark]**

The area of triangle OAB $= \frac{1}{2} \times 15 \times 15 = 112.5$ **[1mark]**

The area of the sector OCE $= \frac{1}{4} \times \pi \times 10^2 = 25\pi$ **[1mark]**

Therefore, the shaded region $= 112.5 - 25\pi = 33.96$ cm^2 **[1mark]**

7 ABCD is a trapezium
A circle is inscribed into the trapezium as shown



AB = 8.3 cm
AD = 6 cm
CD = 5.2 cm
The sides of the trapezium are tangents to the circle
Calculate the shaded area to 2 decimal places.

Solution

The diameter of the circle is the same as the height of the trapezium. That is, AD = 6 cm
So, the radius of the circle will be 3 cm **[1mark]**

{The area of a circle = πr^2}
So, we have A = $\pi \times 3^2 = 9\pi$ **[1mark]**

{The area of a trapezium= $\frac{1}{2}(a+b)h$}
So, we have, A = $\frac{1}{2}(5.2+8.3)\times 6 = 40.5$ **[1mark]**
Hence, the shaded area = $40.5 - 9\pi = 12.23$ cm^2(2dp)
[1mark]