FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving
Area and Perimeter Set 2
Solutions

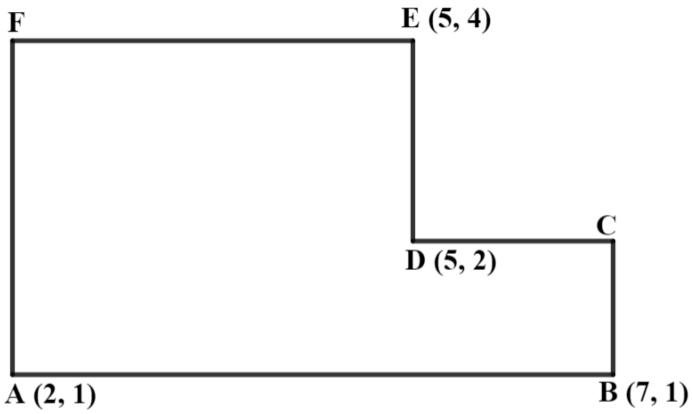
The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1	Here is rectangle ABCD  BC $= \frac{2}{5}$ cm The area of the rectangle is $\frac{4}{7}$ cm^2 Calculate the perimeter of the rectangle Give your answer as a mixed number. **<u>Solution</u>** Length = area ÷ width Therefore, AB $= \frac{4}{7} \div \frac{2}{5}$ $AB = \frac{4}{7} \times \frac{5}{2} = \frac{20}{14} = \frac{10}{7}$ **[1mark]** Therefore, the perimeter of the rectangle will be $\frac{10}{7} + \frac{10}{7} + \frac{2}{5} + \frac{2}{5} = \frac{20}{7} + \frac{4}{5}$ **[2marks]** $P = \frac{20}{7} + \frac{4}{5} = \frac{100}{35} + \frac{28}{35} = \frac{128}{35}$ **[1mark]** {Change your answer to a mixed number} $\frac{128}{35} = 3\frac{23}{35}$ **[1mark]**

2 Here is a 2D shape

Some vertices of the shape are given as coordinates



The coordinates of A are (2, 1)

The coordinates of B are (7, 1)

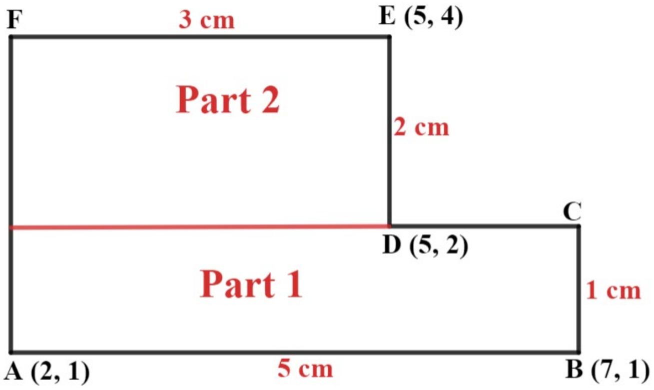
The coordinates of D are (5, 2)

The coordinates of E are (5, 4)

Calculate the area of the shape.

<u>Solution</u>

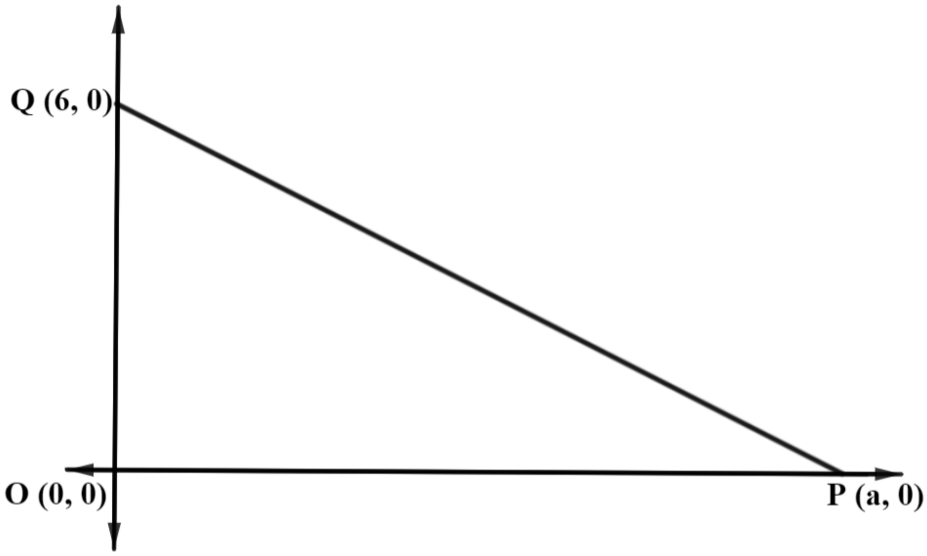
Split the shape into two rectangles



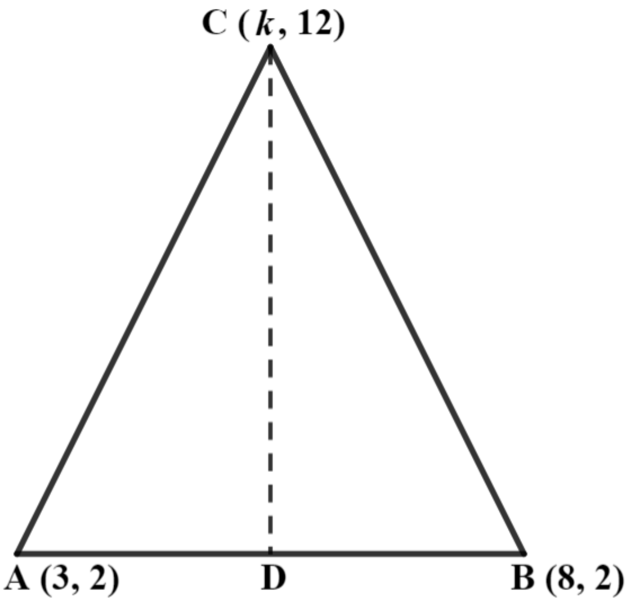
Area of Part 1 $= 5 \times 1 = 5 \text{ cm}^2$ **[2marks]**

Area of Part 2 $= 3 \times 2 = 6 \text{ cm}^2$ **[2marks]**

Area of the shape $= 5 + 6 = 11 \text{ cm}^2$ **[1mark]**

3	PQ is a straight line that intersects the x and y axes at P and Q respectively  O has coordinates (0, 0) Q has coordinates (0, 6) P has coordinates (a, 0) If the area of triangle OPQ is 30 cm^2, find the value of a. **Solution** The area of a triangle $= \frac{1}{2} b \times h$ {Notice that the height of the triangle is 6} The base length of the triangle will be a Therefore, $30 = \frac{1}{2} \times a \times 6$ **[1mark]** $60 = 6a$ $a = 10$ **[1mark]**

4 Triangle ABC is an isosceles triangle



AC = BC
CD is perpendicular to AB
D is the midpoint on AB
A has coordinates (3, 2)
B has coordinates (8, 2)
C has coordinates (k, 12)

a) Write down the value of k

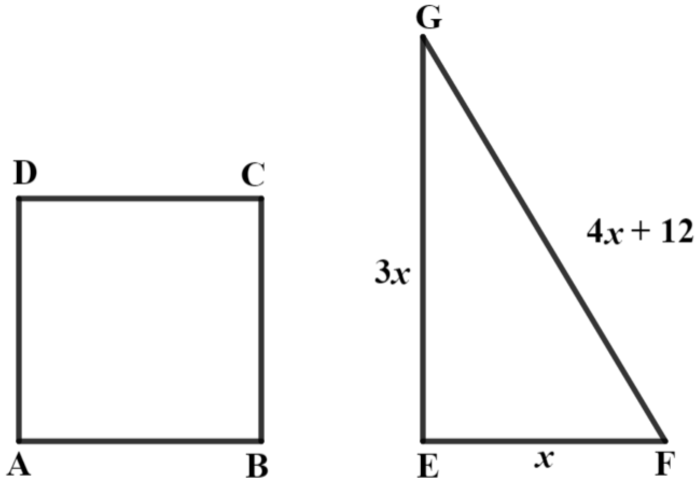
k is the x coordinate of point D
D is the midpoint between A and B
Therefore, $(8 + 3) \div 2 = 11 \div 2 = 5.5$ **[1mark]**
Hence, $k = 5.5$ **[1mark]**

b) Calculate the area of triangle ABC

The base length of the triangle is 8 – 3 = 5 **[1mark]**
The height of the triangle is the y coordinate of C
Therefore, the area of the triangle $= \frac{1}{2} \times 10 \times 5 = 25 \text{ cm}^2$
[2marks]

5

ABCD is a square
EFG is a triangle



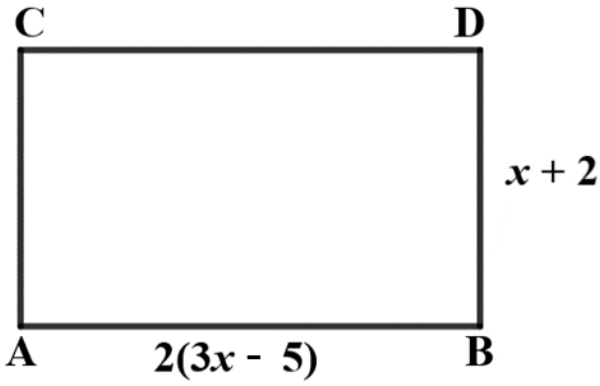
$EG = 3x$
$EF = x$
$FG = 4x + 12$
The area of the square is 144 cm^2
The square and the triangle have the same perimeter.
Calculate the value of x.

<u>Solution</u>

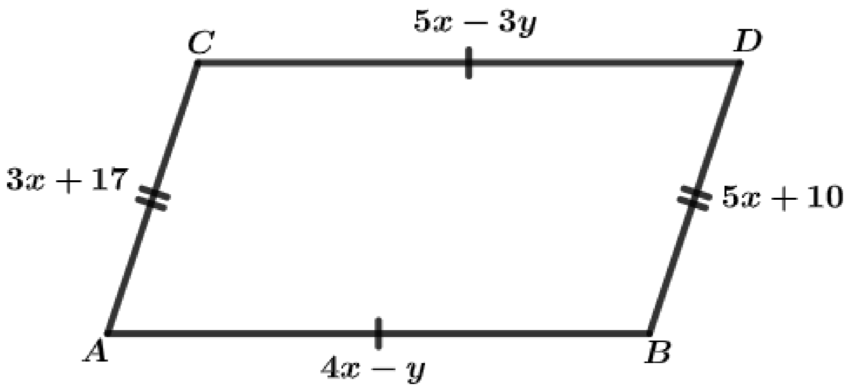
The side length of the square will be $\sqrt{144} = 12$
[1mark]
Therefore, the perimeter of the square will be
$12 \times 4 = 48$ cm **[1mark]**

The perimeter of the triangle will be
$3x + x + 4x + 12 = 8x + 2$

Therefore, $8x + 12 = 48$ **[1mark]**
$8x = 36$
$x = 4.5$ **[1mark]**

6	ABCD is a rectangle  All measurements are in centimeters. AB $= 2(3x - 5)$ BD $= x + 2$ Where x is an integer Given that AB is longer than AC, find the smallest possible area of the rectangle. **Solution** AB is longer than BD Therefore, $2(3x - 5) > x + 2$ $6x - 10 > x + 2$ **[1mark]** $5x > 12$ $x > \frac{12}{5} = 2.4$ **[1mark]** Since x is an integer, it means x is a whole number Therefore, the smallest possible value of x is 3 **[1mark]** So, AB $= 6 \times 3 - 10 = 8$ AC $= 3 + 2 = 5$ **[1mark]** Hence, the smallest possible area of the rectangle is $8 \times 5 = 40$ cm^2 **[1mark]**

7 ABCD shown below is a parallelogram.



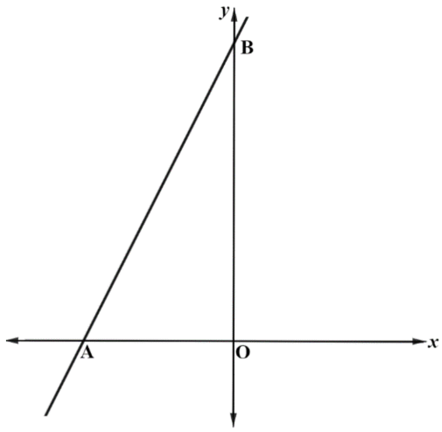
AB = CD and AC = BD
Calculate the values of x and y

Solution

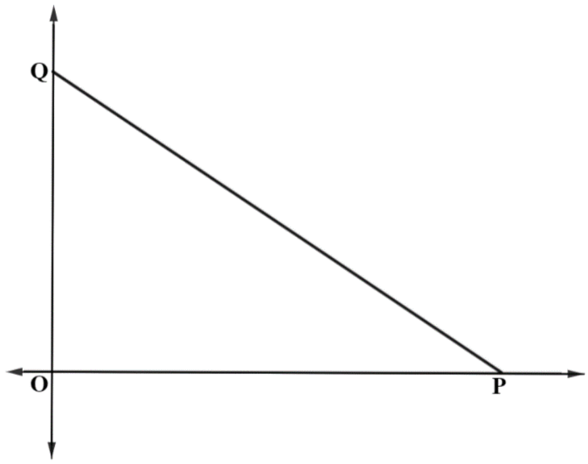
AC = BD
Therefore, $3x + 17 = 5x + 10$
$3x = 5x - 7$
$-2x = -7$
$x = 3.5$ **[2marks]**
AB = CD
Therefore, $4x - y = 5x - 3y$

Substitute $x = 3.5$ into $4x - y = 5x - 3y$

$4 \times 3.5 - y = 5 \times 3.5 - 3y$ **[1mark]**
$14 - y = 17.5 - 3y$
$2y = 3.5$
$y = 1.75$ **[2marks]**

8	The line $y = 2x + 8$ is shown in the diagram below  The line intersects the axes at points A and B Calculate the area of triangle AOB **Solution** When $x = 0$ $y = 2 \times 0 + 8 = 8$ **[1Mark]** The y-intercept is $(0, 8)$ Therefore, B has coordinates $(0, 8)$ So, the height of the triangle will be 8 When $y = 0$ $0 = 2x + 8$ $2x = -8$ $x = -\frac{8}{2} = -4$ **[2marks]** The x −intercept is $(-4, 0)$ Therefore, A has coordinates (-4, 0) Therefore, the area of triangle AOB will be $\frac{1}{2} \times 4 \times 8 = 16$ square units**[1Mark]**

9 The graph of $3y + 2x - 20 = 0$ is shown below



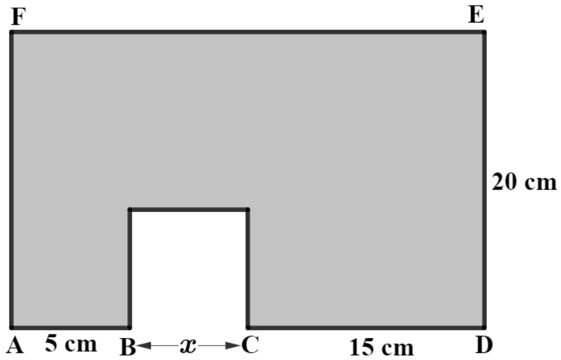
The graph intersects the axes at P and Q
Calculate the area of triangle OPQ
Give your answer as a mixed number.

Solution

When $x = 0$
We have, $3y + 2 \times 0 - 20 = 0$
$3y = 20$
Therefore $y = \frac{20}{3}$ **[1mark]**
So, the height of the triangle is $\frac{20}{3}$
When $y = 0$
We have, $3 \times 0 + 2x - 20 = 0$
$2x = 20$
Therefore, $x = 10$ **[1mark]**
So, the base of the triangle is 10
Therefore, the area of triangle OPQ will be
$\frac{1}{2} \times 10 \times \frac{20}{3} = \frac{100}{3}$ {Change to a mixed number as required}
$\frac{100}{3} = 33\frac{1}{3}$ **[2marks]**

10 ADEF is a rectangle

A square of side x is removed from the rectangle as shown



AB = 5 cm

BC = x

CD = 15 cm

DE = 20 cm

If the shaded area is 484 cm^2,

show that there are two possible values of x

Solution

Length of the rectangle = $15 + 5 + x = 20 + x$

The area of the rectangle = $20(20 + x) = 400 + 20x$ **[1mark]**

The area of the square = x^2

The area of the shaded region = $400 + 20x - x^2$ **[1mark]**

Therefore, $400 + 20x - x^2 = 484$

$x^2 - 20x + 84 = 0$

$(x - 6)(x - 14) = 0$ **[1mark]**

Therefore, $x = 6$ or $x = 14$ **[1mark]**

Check

If $x = 6$ then, $400 + 20 \times 6 - (6)^2 = 484$

If $x = 14$ then, $400 + 20 \times 14 - (14)^2 = 484$ **[1mark]**

Hence, both values of x are valid.