

# FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving

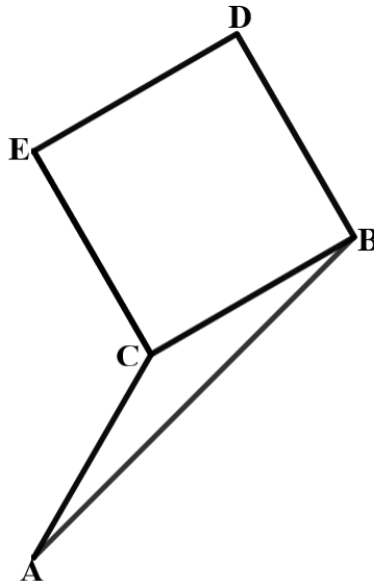
Angles Set 3

Polygons – Solutions

**The marks shown are for guidance purposes only**

**The questions are repeated here for your convenience**

- 1 ABC is an isosceles triangle where  $AC = BC$   
BCDE is a square



Angle ABC is  $15^\circ$

AC and CE are sides of a regular polygon

How many sides does the polygon have?

### Solution

From triangle ABC

$$\text{Angle } ACB = 180 - 15 - 15 = 150$$

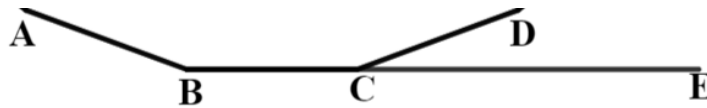
$$\text{Angle } ACE = 360 - 90 - 150 = 120$$

Therefore, an exterior angle of the polygon is

$$180 - 120 = 60 \quad \text{[2marks]}$$

$$\text{Hence, the number of sides is } 360 \div 60 = 6 \quad \text{[1mark]}$$

2 ABCD is part of a regular polygon



BCE is a straight line

The ratio of angle BCD to angle DCE is 9:1

How many sides does the polygon have?

**Solution**

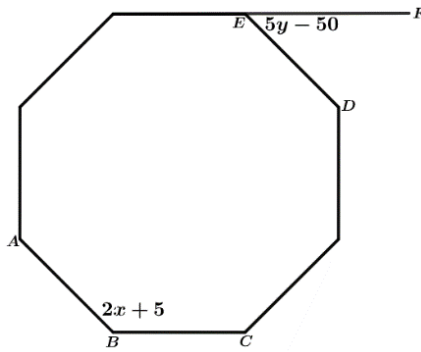
$$180 \div 10 = 18$$

Therefore, angle DCE is  $18^\circ$  [1mark]

$$360 \div 18 = 20$$
 [1mark]

Therefore, the polygon has 20 sides

3 A regular octagon is shown below



$$\text{Angle } ABC = 2x + 5$$

$$\text{Angle } FED = 5y - 50$$

Calculate the values of  $x$  and  $y$

**Solution**

Interior angle of a regular octagon

$$(8 - 2) \times 180 = 6 \times 180 = 1080$$

$$\text{Each interior angle will be } 1080 \div 8 = 135$$

[1mark]

$$\text{So, the exterior angle will be } 180 - 135 = 45$$

[1mark]

Therefore,

$$2x + 5 = 135$$

$$2x = 130$$

$$x = 65$$

[1mark]

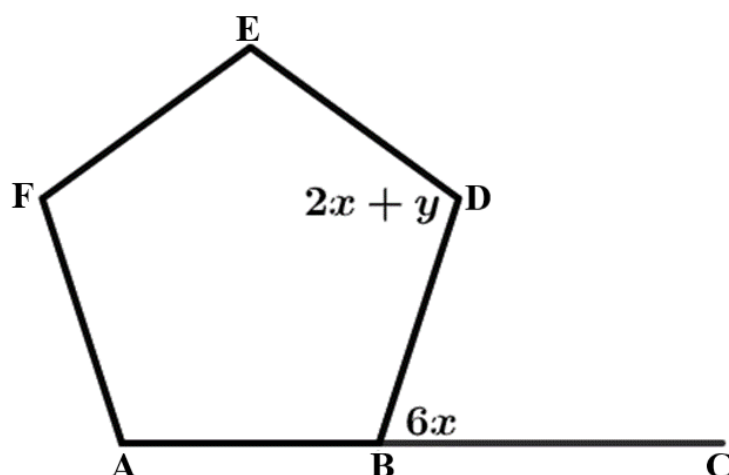
$$\text{Also, } 5y - 50 = 45$$

$$5y = 95$$

$$y = 19$$

[1mark]

4 ABDEF is a regular pentagon



$$\text{Angle BDE} = 2x + y$$

$$\text{Angle CBD} = 6x$$

ABC is a straight line

Calculate the values of  $x$  and  $y$ .

**Solution**

$$\text{Sum of interior angles of a polygon} = (n - 2) \times 180$$

Therefore, for a pentagon we have

$$(5 - 2) \times 180 = 3 \times 180 = 540$$

$$\text{Each interior angle is therefore } 540 \div 5 = 108 \text{ [1mark]}$$

$$\text{Hence, } 2x + y = 108$$

$$\text{Also Angle CAB} = 72$$

{ $180 - 108 = 72$ . Angles at a point on a straight line}

$$\text{Therefore, } 6x = 72 \quad \text{[1mark]}$$

$$x = 12 \quad \text{[1mark]}$$

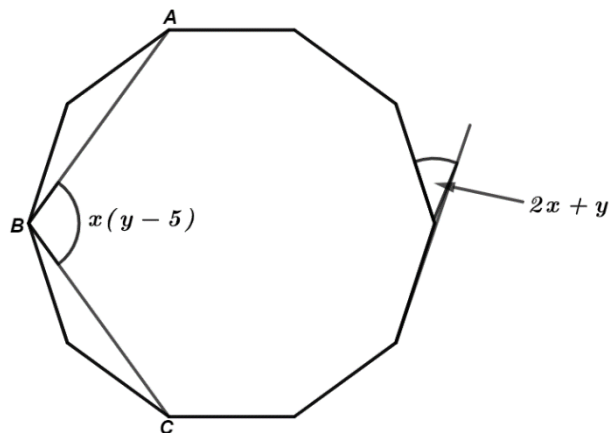
Therefore,

$$2 \times 12 + y = 108$$

$$24 + y = 108 \quad \text{[1mark]}$$

$$y = 84 \quad \text{[1mark]}$$

5 The diagram below shows a regular decagon.



$$\text{Angle } ABC = x(y - 5)$$

The exterior angle of the decagon is  $2x + y$

Find the values of  $x$  and  $y$

Give your answers to 3 significant figures

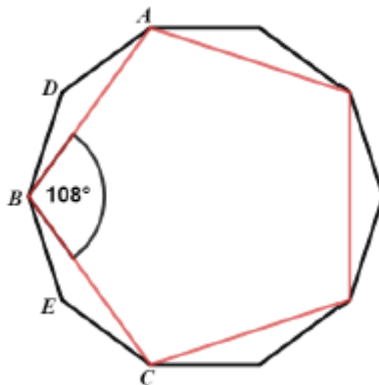
### Solution

An exterior angle of decagon =  $360 \div 10 = 36$

{Note: the sum of the exterior angles of any polygon is  $360^\circ$ }

Therefore,  $2x + y = 36$  [1mark]

Now find the size of the angle ABC as follows:



From the diagram above, you notice that the angle ABC is an interior angle of a regular pentagon.

Sum of the angles in a pentagon =  $3 \times 180 = 540$

Therefore, each angle  $540 \div 5 = 108^\circ$

Therefore,  $x(y - 5) = 108$  [1mark]

So, we now have simultaneous equations

$$2x + y = 36$$

$$x(y - 5) = 108 \text{ -----(1)}$$

From the first equation

$$2x + y = 36$$

Therefore,  $y = 36 - 2x$

Now substitute this for  $y = 36 - 2x$  in (1)

$$x(36 - 2x - 5) = 108 \quad [1mark]$$

$$x(31 - 2x) = 108$$

$$31x - 2x^2 = 108$$

$$-2x^2 + 31x - 108 = 0 \quad [1mark]$$

Solve using the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$a = -2; b = 31 \text{ and } c = -108$$

$$x = \frac{-31 \pm \sqrt{31^2 - 4(-2)(-108)}}{2(-2)}$$

$$x = 10.2 \text{ or } x = 5.29 \quad [1mark]$$

$$\text{Hence, } y = 36 - 2(10.2) = 15.6$$

Or

$$y = 36 - 2(5.29) = 25.4$$

[1mark]