FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving
Quadratic Sequences
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 The Nth term of a quadratic sequence is given as

$$n^2 + an + b$$

The 3rd and 4th terms of the sequence are 17 and 27 respectively. Find the value of the 10th term.

Solution

Find the 3rd and 4th terms in terms of a and b

The 3rd term is $(3)^2 + a(3) + b = 9 + 3a + b$

So $9 + 3a + b = 17$

Therefore, $3a + b = 8$ ----------(1) **[1mark]**

The 4th term is $(4)^2 + a(4) + b = 16 + 4a + b$

So $16 + 4a + b = 27$

Therefore, $4a + b = 11$---------- (2) **[1mark]**

Subtract equations 1 and 2

$3a + b = 8$

$\underline{4a + b = 11}$

$a = 3$ **[1mark]**

Substitute $a = 3$ into equation 1

$3(3) + b = 9$

Therefore, $b = -1$ **[1mark]**

Therefore, the Nth term is $n^2 + 3n - 1$

Hence, the 10th term is $(10)^2 + 3(10) - 1 = 129$ **[1mark]**

2 Here are the first 5 terms of a quadratic sequence

1.5, 12, 26.5, 45, 67.5

Find the difference between the 30th and 40th terms

Solution

Find the Nth term of the sequence

Sequence: 1.5, 12, 26.5, 45, 67.5

1st difference: 10.5, 14.5, 18.5, 22.5

2nd difference: 4, 4, 4

Therefore, the sequence has a $2n^2$

Sequence: 1.5, 12, 26.5, 45, 67.5

$2n^2$: 2, 8, 18, 32, 50

Difference: $-0.5, 4, 8.5, 13, 17.5$

The Nth term of the difference is $4.5n - 5$

Hence, the Nth term of the quadratic sequence is

$2n^2 + 4.5n - 5$ **[3marks]**

The 30th term is $2(30)^2 + 4.5(30) - 5 = 1930$

The 40th term is $2(40)^2 + 4.5(30) - 5 = 3375$

[1mark]

$3375 - 1930 = 1445$ **[1mark]**

3 The Nth term of a quadratic sequence is $an^2 + 5n - 7$
The difference between the 3rd and 2nd terms is -10
Find the 100th term of the sequence

Solution

The 2nd term is $a(2)^2 + 5(2) - 7 = 4a + 3$ **[1mark]**

The 3rd term is $a(3)^2 + 5(3) - 7 = 9a + 8$ **[1mark]**

Therefore, $(9a + 8) - (4a + 3) = -10$
$5a + 5 = -10$
$5a = -15$
$a = -3$ **[1mark]**
Therefore, the Nth term is $-3n^2 + 5n - 7$

Hence, the 100th term is
$-3(100)^2 + 5(100) - 7 = -29507$ **[1mark]**

4 The Nth term of a quadratic sequence is $n^2 - kn + 42$
The 2nd term in the sequence is 62
Find the smallest possible term in the sequence.

Solution

$(2)^2 - k(2) + 42 = 62$ **[1mark]**
$-2k = 16$
$k = -8$ **[1mark]**
Therefore, the Nth of the sequence is $n^2 - 8n + 42$
The smallest term is required, therefore, write the expression in completed square form $n^2 - 8n + 42 = (n - 4)^2 - 16 + 42$ **[1mark]**
$(n - 4)^2 + 26$ the minimum point is $(4, 26)$ **[1mark]**
Hence, the smallest term is 26 **[1mark]**

5 Here are the first 5 hexagonal numbers

$$1, 6, 15, 28, 45, \ldots$$

The Nth term of the sequence is $an^2 + bn$

Find the values of the 10th hexagonal number

Solution

$a(1)^2 + b(1) = 1$

$a + b = 1$ ----------equation 1 **[1mark]**

$a(2)^2 + b(2) = 6$

$4a + 2b = 6$ {Divide both sides by 2}

$2a + b = 3$ ----------equation 2 **[1mark]**

Subtract equation 1 from equation 2

$2a + b = 3$

$\underline{a + b = 1}$

$a = 2$ **[1mark]**

Therefore, $2 + b = 1$

$b = -1$ **[1mark]**

Therefore, the Nth term of the sequence is $2n^2 - n$

Hence, the 10th term is $2(10)^2 - (10) = 190$ **[1mark]**