FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving
Non-linear Sequences
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1

The Nth term of a sequence is given as $\left(\frac{5n}{6}\right)^2$

The 4th term of the sequence is $7x$

Find the value of x

Give your answer as a mixed number.

Solution

Substitute $n = 4$ into the Nth term formula

$\left(\frac{5\times4}{6}\right)^2 = \left(\frac{20}{6}\right)^2 = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$ **[1mark]**

Therefore, $7x = \frac{100}{9}$ **[1mark]**

$63x = 100$

$x = \frac{100}{63} = 1\frac{37}{63}$ **[1mark]**

2

The Nth term of a sequence is given as $5n + 3^n$

Find the sum of the first 3 terms.

Solution

The first term is $5 \times 1 + 3^1 = 8$ **[1mark]**

The second term is $5 \times 2 + 3^2 = 19$ **[1mark]**

The third term is $5 \times 3 + 3^3 = 42$ **[1mark]**

Therefore, the sum of the first 3 terms is

$8 + 19 + 42 = 69$ **[1mark]**

3 The first 4 terms of a Fibonacci sequence are given below

$$3p,\ 2q,\ 3p+2q,\ 3p+4q,\ldots$$

Given that the 3rd and 4th terms are 15 and 31 respectively, find the values of p and q

Solution

$3p + 2q = 15$ ----------Equation 1

$3p + 4q = 31$ ----------Equation 2

Subtract equation 1 from equation 2

$2q = 16$

$q = 8$ **[1mark]**

Now substitute $q = 8$ into equation 1

$3p + 2(8) = 15$ **[1mark]**

$3p + 16 = 15$ {Multiply through by 3}

$3p = -1$

$p = -\frac{1}{3}$ **[2marks]**

4 The Nth term of a sequence is given as $5n + 3^{n-p}$

The 7th term of the sequence is 62.

Find the value of p.

Solution

Substitute $n = 7$ into the Nth term expression

That is, $5 \times 7 + 3^{7-p} = 62$ **[1mark]**

$35 + 3^{7-p} = 62$

$3^{7-p} = 27$ {Write 27 into powers on 3} **[1mark]**

$3^{7-p} = 3^3$ **[1mark]**

Therefore, $7 - p = 3$

$p = 4$ **[1mark]**

5 Here are the first three terms of a certain sequence.

$$\frac{1}{5a}, \frac{2}{3a+b}, \frac{3}{a+2b}, \ldots$$

The 2nd and 4th terms of the sequence are $\frac{1}{3}$ and 1 respectively. Calculate the values of a and b.

Solution

The term to term rule of the numerator is $+1$

The term to term rule of the denominator is $-2a + b$

Hence, the fourth term is $\frac{4}{-a+3b}$

Therefore, we have $\frac{2}{3a+b} = \frac{1}{3}$

$6 = 3a + b$ ----------Equation 1 **[1mark]**

Therefore, $b = 6 - 3a$

Similarly $\frac{4}{-a+3b} = 1$ {Cross multiply}

$4 = -a + 3b$ ----------Equation 2 **[1mark]**

From equation 1

Substitute $b = 6 - 3a$ into equation 2

$4 = -a + 3(6 - 3a)$

$4 = -a + 18 - 9a$

$-14 = -10a$

$a = \frac{14}{10} = \frac{7}{5}$ **[2marks]**

Therefore, $b = 6 - 3 \times \frac{7}{5} = \frac{9}{5}$ **[1mark]**

6 The first 4 terms of a Fibonacci sequence are given below

$$3x, \quad 4y, \quad 3x + 4y, \quad 3x + 8y \ldots$$

The 1st and 5th terms of the sequence are 5 and 20 respectively. Find the ratio of $x : y$ in its simplest form.

Solution

$3x = 5$

Therefore, $x = \frac{5}{3}$ **[1mark]**

The 5th term is $3x + 4y + 3x + 8y = 6x + 12y$

Therefore, $6x + 12y = 20$ **[1mark]**

Substitute $x = \frac{5}{3}$ into $6x + 12y = 20$ **[1mark]**

We have, $6 \times \frac{5}{3} + 12y = 20$

$10 + 12y = 20$

$12y = 10$

Therefore, $y = \frac{10}{12}$ **[1mark]**

So, $x : y = \frac{5}{3} : \frac{10}{12}$

$$= \frac{20}{12} : \frac{10}{12}$$

Hence, $x : y = 2 : 1$ **[1mark]**

7 The first 3 terms of a Fibonacci sequence are given below

$$\frac{1}{2}x, \quad \frac{1}{2}y, \quad \frac{x+y}{2}, \ldots$$

The 4th term in the sequence is 3

Given that $y = 3x$, find the values of x and y

Solutions

The 4th term is $\frac{1}{2}y + \frac{x+y}{2} = \frac{x+2y}{2}$

Therefore, $\frac{x+2y}{2} = 3$ **[1mark]**

$x + 2y = 6$ **[1mark]**

Now substitute $y = 3x$ into $x + 2y = 6$

So, $x + 2 \times 3x = 6$ **[1mark]**

$x + 6x = 6$

$7x = 6$

$x = \frac{6}{7}$ **[1mark]**

Therefore, $y = 3 \times \frac{6}{7} = \frac{18}{7}$ **[1mark]**