FluidMaths

GCSE Mathematics (Grade 9-1)

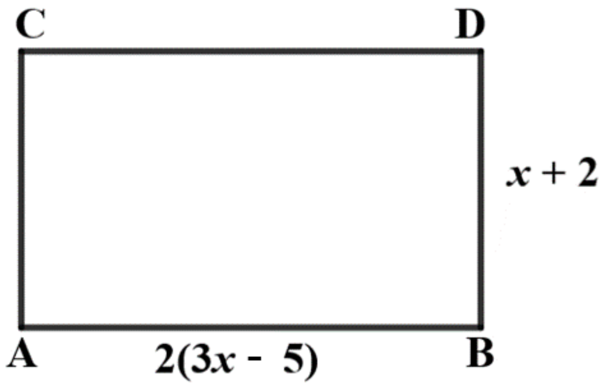
Problem Solving
Linear Inequalities
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 ABCD is a rectangle

All measurements are in centimeters.



$AB = 2(3x - 5)$

$BD = x + 2$

Where x is an integer

Given that AB is longer than AC,

find the smallest possible area of the rectangle.

Solution

AB is longer than BD

Therefore, $2(3x - 5) > x + 2$

$6x - 10 > x + 2$ **[1mark]**

$5x > 12$

$x > \frac{12}{5} = 2.4$ **[1mark]**

Since x is an integer, it means x is a whole number

Therefore, the smallest possible value of x is 3 **[1mark]**

So, $AB = 6 \times 3 - 10 = 8$

$AC = 3 + 2 = 5$ **[1mark]**

Hence, the smallest possible area of the rectangle is

$8 \times 5 = 40\ cm^2$ **[1mark]**

2 Here are the charges for two taxi companies

Company A	**Company B**
£15 + £6 per hour	£25 + £4 per hour

Oscar wants to hire a taxi for some hours.
What is the least number of hours that makes it better to hire from Company B?

Solution

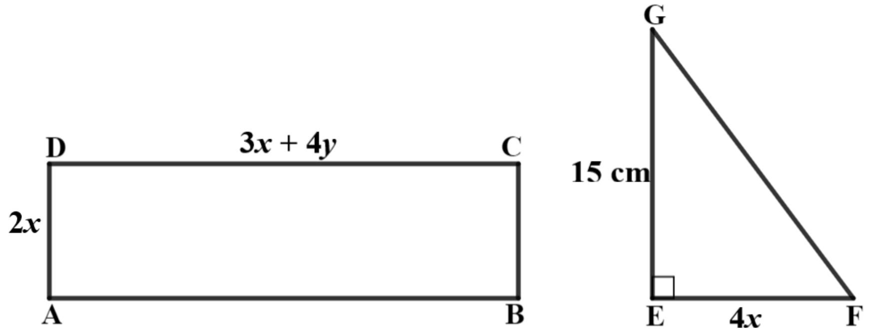
Let x be the number of hours Oscar needs the taxi for
Then from company A, his cost will be $6x + 15$
From company B his cost will be $4x + 25$ **[1Mark]**

Since we want the price to be better from company B then
$4x + 25 < 6x + 15$ **[1Mark]**
$-2x < -10$
$x > 5$ **[1Mark]**

Therefore, Oscar needs to hire a taxi for more than 5 hours for company B to be of better value.

3	Ben is a car valet. He charges £25 per valet Ben spends £65 a day to pay for his expenses. Ben wants to make a daily profit of at least £135 but not more than £185. How many cars should Ben aim to valet a day? **Solution** This is a problem involving inequalities Let x be the number of cars Ben needs to valet per day. Then Ben's profits will follow the inequality, $135 \leq 25x - 65 \leq 185$[**1Mark**] Now solve the above inequality to find x $135 \leq 25x - 65 \leq 185$ $200 \leq 25x \leq 250$ **[1Mark]** $8 \leq x \leq 10$ **[1Mark]** Ben should aim to valet between 8 and 10 cars a day.
4	Given that $x: y = 2: 3$ and $6x + 21y < 70$, Find the largest integer value of y. **Solution** $\frac{x}{y} = \frac{2}{3}$ therefore, $x = \frac{2}{3}y$ **[1mark]** Substitute $x = \frac{2}{3}y$ into $6x + 21y < 70$ $6\left(\frac{2}{3}y\right) + 21y < 70$ $4y + 21y < 70$ **[1mark]** $25y < 70$ $y < 2.8$ **[1mark]** Hence, the largest integer value of y is 2 **[1mark]**

5 ABCD is a rectangle
EFG is a triangle
AD $= 2x$
CD $= 3x + 4y$
EF $= 4x$
EG $= 15$ cm



The perimeter of the rectangle is known to be larger than the perimeter of the triangle.
What is the least integer value of y?

<u>Solution</u>

$\frac{15 \times 4x}{2} = 60$

$60x = 120$

$x = 2$ **[1mark]**

Therefore, EF $= 8$ cm

Use Pythagoras theorem to find FG

$(\text{FG})^2 = 15^2 + 8^2$

$\text{FG} = \sqrt{289} = 17$ **[1mark]**

Therefore, the perimeter of the triangle is $17 + 15 + 8 = 40$

The perimeter of the rectangle is

$2x + 2x + 3x + 4y + 3x + 4y > 40$ **[1mark]**

$10x + 8y > 40$

Substitute $x = 2$ into $10x + 8y > 40$

$10(2) + 8y > 40$ **[1mark]**

$20 + 8y > 40$

$8y > 20$

$y > 2.5$ **[1mark]**

Hence, the least integer value of y is 3 **[1mark]**

6	Given that $3x\sqrt{5}+\sqrt{12}\geq x\sqrt{125}$, prove that $x\geq\frac{\sqrt{15}}{5}$ **<u>Solution</u>** $3x\sqrt{5}-x\sqrt{125}\geq-\sqrt{12}$ $3x\sqrt{5}-5x\sqrt{5}\geq-2\sqrt{3}$ **[1mark]** $-2x\sqrt{5}\geq-2\sqrt{3}$ **[1mark]** $x\geq\frac{\sqrt{3}}{\sqrt{5}}$ $x\geq\frac{\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$ **[1mark]** Hence, $x\geq\frac{\sqrt{15}}{5}$ **[1mark]**
7	Solve the equality below $3^4\leq3^{5-2x}\leq243$ **<u>Solution</u>** Write $3^4\leq3^{5-2x}\leq243$ as powers of 3 $3^4\leq3^{5-2x}\leq3^5$ **[1mark]** Therefore, $4\leq5-2x\leq5$ {Subtract 5 from both sides} **[1mark]** $-1\leq-2x\leq0$ {Divide both sides by -2} **[1mark]** $\frac{1}{2}\geq x\geq0$ **[1mark]**

8	Given that $5^{8x} \leq 125^{y-5}$ and $27^{x} = \frac{1}{81}$ Calculate the least integer value of y Solution Write $27^{x} = \frac{1}{81}$ as powers of 3 $3^{3x} = 3^{-4}$ Therefore, $3x = -4$ Hence, $x = -\frac{4}{3}$ **[1mark]** Write $5^{8x} \leq 125^{y-5}$ as powers of 5 $5^{8x} \leq 5^{3(y-5)}$ **[1mark]** Therefore, $8x \leq 3y - 15$ **[1mark]** Now substitute $x = -\frac{4}{3}$ into $8x \leq 3y - 15$ $8\left(-\frac{4}{3}\right) \leq 3y - 15$ $-\frac{32}{3} \leq 3y - 45$ **[1mark]** $-32 \leq 9y - 45$ $13 \leq 9y$ $y \geq \frac{13}{9}$ **[1mark]** $y \geq 1.44$ (3sf) Hence, the least integer value of y is 2 **[1mark]**