FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving
Functions
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 Here are three functions

$f(x) = 2ax + b$;

$g(x) = 4x^2 - 12x - 7$ and

$h(x) = 2x + 1$

Given that $\frac{g(x)}{h(x)} = f(x)$,

find the values of a and b.

Solution

$\frac{g(x)}{h(x)} = \frac{4x^2-12x-7}{2x+1}$

{Factorise the quadratic numerator and simplify}

$\frac{g(x)}{h(x)} = \frac{\cancel{(2x+1)}(2x-7)}{\cancel{2x+1}}$ {Cancel out the common factors}**[1mark]**

Therefore, $\frac{g(x)}{h(x)} = 2x - 7$ **[1mark]**

Hence, $2x - 7 = 2ax + b$

Therefore, $2a = 2$ so

$a = 1$ and $b = -7$

[2marks]

2 Two functions are such that
$g(x) = 20 - kx$; and
$h(x) = -3k - 7$
Given that $gh(x) = 33$ find the possible values of k.
Give your answers to 2 decimal places.

Solution

$gh(x)$ means substitute $h(x)$ into $g(x)$
$gh(x) = 20 - k(-3k - 7)$
{Expand the brackets and simplify}
$gh(x) = 20 + 3k^2 + 7k$ **[1mark]**
Therefore, $3k^2 + 7k + 20 = 33$
{Subtract 33 from both sides}
Hence, $3k^2 + 7k - 13 = 0$ **[1mark]**

Solve using the quadratic formula
$a = 3, b = 7$ and $c = -13$

$k = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times -13}}{2 \times 3}$ **[1mark]**

Therefore, $k = 1.22$ or $k = -3.55$ (2dp)
[2marks]

3 The equation of a quadratic function is given as

$$f(x) = k(x+b)^2 + c$$

The minimum point of the function is $(1, -7)$

The y-intercept of the function is $(0, -5)$

a) Find the values of k, b and c.

b) Hence write $f(x)$ in the form $f(x) = ax^2 + bx + c$

Solution

a) Since the minimum point is $(1, -7)$

then $b = -1$ and $c = -7$ **[2marks]**

Therefore, $f(x) = k(x-1)^2 - 7$

The y-intercept is $(0, -5)$

Substitute this into the equation $y = k(x-1)^2 - 7$

$-5 = k(0-1)^2 - 7$

$-5 = k - 7$

$k = 2$ **[2marks]**

Hence, the required values are $k = 2$, $b = -1$ and $c = -7$

b) $f(x) = 2(x-1)^2 - 7$ {Expand the brackets}

$= 2(x^2 - 2x + 1) - 7$ **[1mark]**

$= 2x^2 - 4x + 2 - 7$

$= 2x^2 - 4x - 5$ **[1mark]**

4	A function is defined as $\mathrm{f}(x) = ax^2 + 6x + c$ A graph of the function passes through $(-8, 15)$ and $(0, -1)$ Write $\mathrm{f}(x)$ in the form $(x + p)^2 + q$, Stating the values of p and q **Solution** The y-intercept for any function is the point where $x = 0$ Therefore, $c = -1$ **[1mark]** So, $\mathrm{f}(x) = ax^2 + 6x - 1$ Now substitute the coordinate $(-8, 15)$ into the function $15 = a(-8)^2 + 6(-8) - 1$ $15 = 64a - 49$ $64a = 64$ $a = 1$ **[1mark]** Hence, $\mathrm{f}(x) = x^2 + 6x - 1$ Write $\mathrm{f}(x)$ in completed square form $\mathrm{f}(x) = (x + 3)^2 - 9 - 1$ **[1mark]** $\mathrm{f}(x) = (x + 3)^2 - 10$ **[1mark]** Therefore $p = 3$ and $q = -10$ **[1mark]**

5	Given that $f^{-1}(x) = \frac{x+2}{3}$, Solve $f(x) = -1$ **Solution** Find $f(x)$ from the given inverse {Swap x and y in the inverse function, and make y the subject} $x = \frac{y+2}{3}$ **[1mark]** $3x = y + 2$ $y = 3x - 2$ **[1mark]** Therefore, $f(x) = 3x - 2$ So, $3x - 2 = -1$ **[1mark]** $3x = 1$ $x = \frac{1}{3}$ **[1mark]**
6	Given that, $f(x) = 5x + 3$ Solve $f^{-1}f(x) = 7 - 3x$. **Solution** Find the inverse of $f(x)$ Step 1: replace $f(x)$ with y. That is $y = 5x + 3$ Step 2: switch x and y around. That is $x = 5y + 3$ Step 3: make y the subject. That is $\frac{x-3}{5} = y$ Therefore, $f^{-1}(x) = \frac{x-3}{5}$ **[1mark]** To find $f^{-1}f(x)$, substitute $f(x)$ into f^{-1} So, $f^{-1}f(x) = \frac{5x+3-3}{5} = \frac{5x}{5}$ Hence, $f^{-1}f(x) = x$ **[2marks]** Therefore, $x = 7 - 3x$ **[1mark]** $4x = 7$ $x = \frac{7}{4}$ **[1mark]**

7	Here are two functions: $g(x) = 3x^2$ and $h(x) = kx - 7$ Given that $hg(x) = 17$, find k in terms of x. **Solution** Find the composite function $hg(x)$ $hg(x)$ means substitute $g(x)$ into $h(x)$ That is, $hg(x) = k(3x^2) - 7$ $= 3kx^2 - 7$ **[1mark]** Therefore, $3kx^2 - 7 = 17$ $3kx^2 = 24$ **[1mark]** $k = \frac{24}{3x^2} = \frac{8}{x^2}$ **[1mark]**
8	Two functions are such that $\mathrm{f}(x) = 13 - 4x$ and $g(x) = ax + 5$ Given that $\mathrm{f}g(x) = 10x - 7$, find the value of a. **Solution** $\mathrm{f}g(x)$ means, substitute $g(x)$ into $\mathrm{f}(x)$ That is: $\mathrm{f}g(x) = 13 - 4(ax + 5)$ **[1mark]** $\mathrm{f}g(x) = 13 - 4ax - 20$ $\mathrm{f}g(x) = -4ax - 7$ **[1mark]** Now compare the coefficients in the above expression with the given expression for $\mathrm{f}g(x)$ $\mathrm{f}g(x) = 10x - 7$ $\mathrm{f}g(x) = -4ax - 7$ therefore, $-4a = 10$ **[1mark]** hence, $a = -2.5$ **[1mark]**