FluidMaths

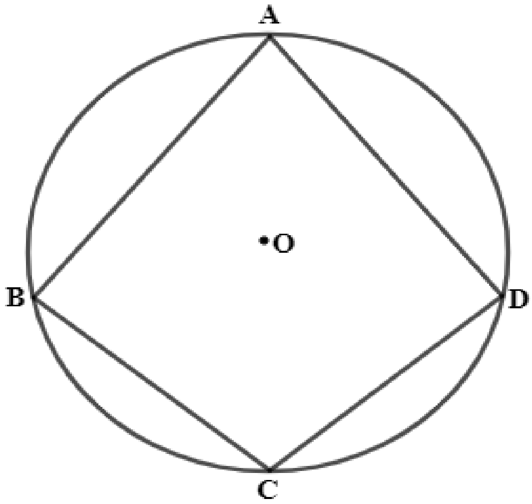
GCSE Mathematics (Grade 9-1)

Problem Solving
Equation of a Circle
Solutions

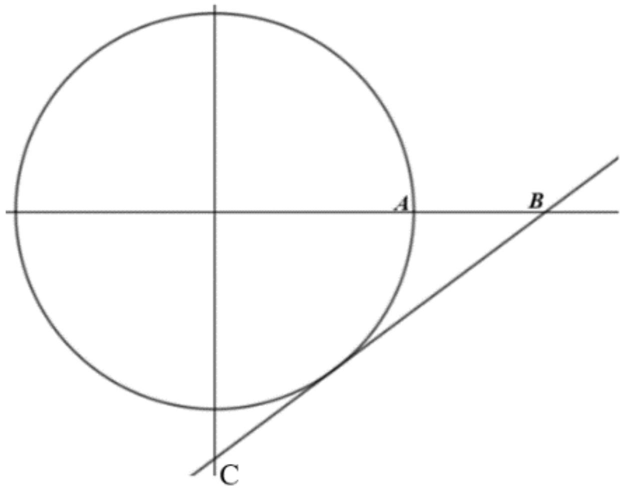
The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1	The area of a circle is given as $48k\pi$ cm^2 Where k is a positive whole number. The centre of the circle is (0, 0) Find the equation of the circle in terms of k. **<u>Solution</u>** The area of a circle $= \pi r^2$ Therefore, $48k\pi = \pi r^2$ So $r^2 = 48k$ **[1mark]** The equation of a circle with centre at (0, 0) is given as $x^2 + y^2 = r^2$ Therefore, the equation of the circle in terms of k is $x^2 + y^2 = 48k$ **[1mark]**
2	The equation of a circle is $x^2 + y^2 = 16a$ Given that the circumference of the circle is 18π cm, calculate the exact value of a. **<u>Solution</u>** {The circumference of a circle $= 2\pi r$} Therefore, $18\pi = 2\pi r$ So, $r = 9$ **[1mark]** The equation of a circle with its centre at the origin O is $\{x^2 + y^2 = r^2\}$ Therefore, for this circle $r^2 = 9^2 = 81$ **[1mark]** So, $16a = 81$ **[1mark]** Hence, $a = \frac{81}{16}$ **[1mark]**

3	ABCD is a kite of area 24 cm². The kite is drawn inside a circle with equation $x^2 + y^2 = 16$  Show that the ratio AC: BD = 4:3 **Solution** The equation of a circle $\{x^2 + y^2 = r^2\}$ By comparing, we can find that $r^2 = 16$ Therefore, $r = 4$. The radius of the circle is 4 cm. Therefore, its diameter will be 8 cm **[1mark]** Hence, line AC = 8 cm. $\left\{\text{Area} = \frac{\text{pq}}{2} \text{ where p and q are the diagonals of the kite}\right\}$ Therefore $24 = \frac{8\times q}{2}$ **[1mark]** $48 = 8q$ $q = 6$ **[1mark]** Thus, the diagonal BD = 6 cm So, the required ratio will be $8{:}\,6 = 4 : 3$ **[1mark]**

4 A circle with centre at (0, 0) is shown below



The equation of the circle is $x^2 + y^2 = 16$

The circle intersects the x-axis at point A

The line BC is a tangent with the equation $y = \frac{3}{4}x - 5$

How far is B from A?

Give your answer to 1 decimal place.

Solution

The centre of the circle is (0,0)

Therefore, $r^2 = 16$

So, r = 4 cm

Therefore, A is the coordinate (4, 0) **[1mark]**

B is the x-intercept of the line.

The x-intercept is the point when $y = 0$

Therefore, $0 = \frac{3}{4}x - 5$

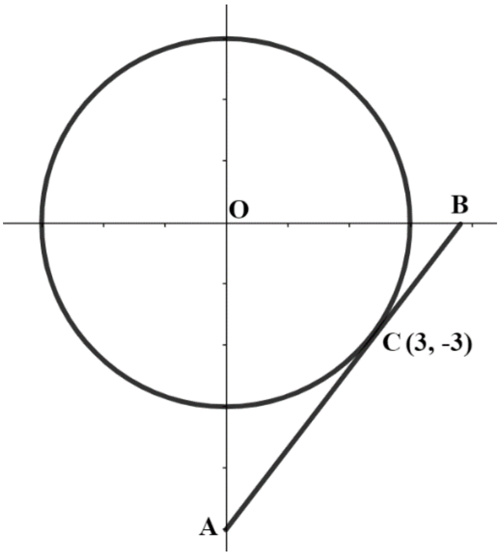
$0 = 3x - 20$

$3x = 20$

$x = \frac{20}{3}$ **[1mark]**

Hence, AB $= \frac{20}{3} - 4 = \frac{8}{3} = 2.7$ (1dp) **[1mark]**

5 The diagram shows a circle with centre at the point O (0,0)



The line AB is a tangent to the circle at C $(3, -3)$
Calculate the distance AO.

Solution

The distance AO is between the y-intercept of the tangent and the origin $(0, 0)$

The gradient of the radius is $m = \frac{0--3}{0-3} = \frac{3}{-3} = -1$ **[1mark]**

The radius and the tangent intersect at right angles

{Angle between a radius and a tangent is 90°}

Therefore, the gradient of the tangent will be 1. **[1mark]**

{Gradient of perpendicular lines have a product of -1}

Therefore, the tangent will have the equation $y = x + c$

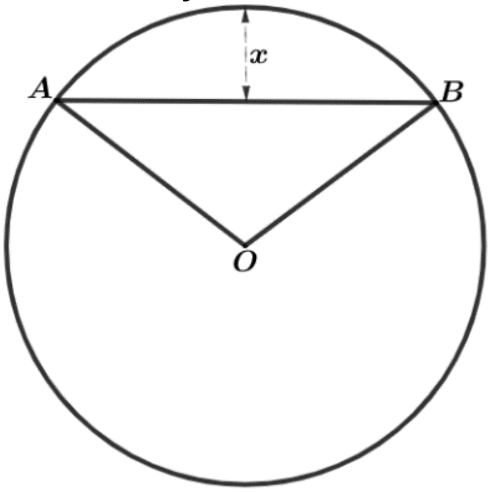
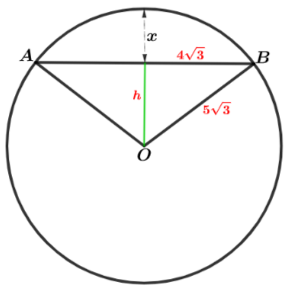
Now substitute $(3, -3)$ into $y = x + c$

$-3 = 3 + c$

Therefore, $c = -6$ **[1mark]**

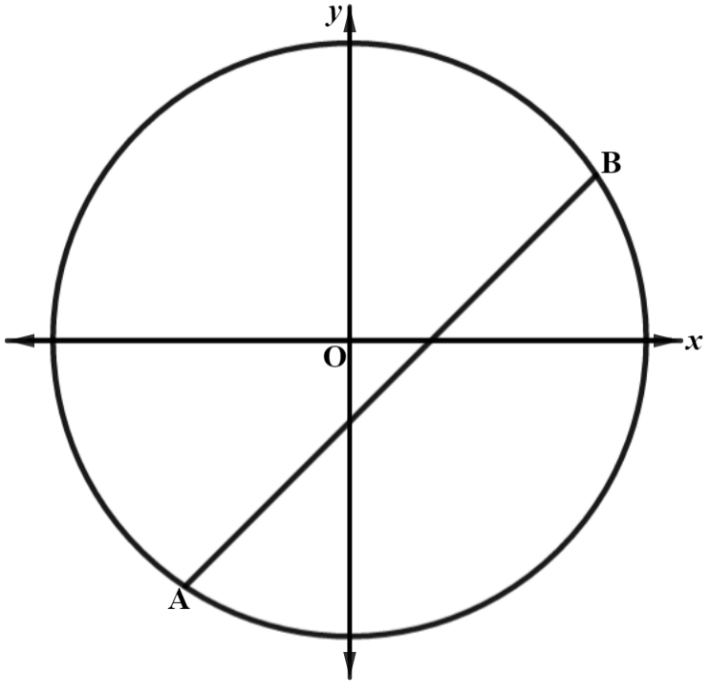
So, the y-intercept of the tangent will be $(0, -6)$

Therefore, $\text{AO} = 6$

6	A circle with equation $x^2 + y^2 - 75 = 0$ is shown below  $\text{AB} = 8\sqrt{3}$ cm Show that $x = 2\sqrt{3}$ **Solution** x is the difference between the radius of the circle and the height of the triangle. Note that AO and OB are both radii of the circle. Equation of a circle with centre at O : $x^2 + y^2 = r^2$ $x^2 + y^2 - 75 = 0$ $x^2 + y^2 = 75$ Therefore, $r^2 = 75$ So $r = \sqrt{75} = 5\sqrt{3}$ **[1mark]** Therefore, $\text{OB} = \text{OA} = 5\sqrt{3}$ Consider the diagram on the RHS Apply Pythagoras theorem $h^2 = (5\sqrt{3})^2 - (4\sqrt{3})^2$ **[1mark]** $h^2 = 75 - 48 = 27$ $h = \sqrt{27} = 3\sqrt{3}$ **[1mark]**  Therefore, the height of the triangle is $3\sqrt{3}$ Hence $x = 5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}$ **[2marks]**

7	The equation of a circle is given as $x^2 + y^2 = 32$ The equation of a tangent to the circle is $y = ax + b$ The tangent touches the circle at point P The coordinates of P are $(p, -5p)$ a) Show that $a = \frac{1}{5}$ b) Show that $b = -\frac{26}{5}p$ c) Given that $b = -\frac{39}{5}$, find the value of p **<u>Solution</u>** a) Coordinates of the centre of the circle are $(0, 0)$ The gradient of the radius will be $\frac{-5p-0}{p-0} = \frac{-5p}{p} = -5$ **[1mark]** Therefore, the gradient of the tangent is $\frac{1}{5}$ {The tangent and the radius are perpendiculars} Hence, $a = \frac{1}{5}$ **[1mark]** b) The equation of the tangent is $y = \frac{1}{5}x + b$ Substitute $a = \frac{1}{5}, x = p$ and $y = -5p$ into $y = \frac{1}{5}x + b$ Therefore, $-5p = \frac{1}{5} \times p + b$ **[1mark]** $-5p = \frac{p}{5} + b$ $b = -5p - \frac{p}{5}$ **[1mark]** Hence, $b = -\frac{26}{5}p$ **[1mark]** c) $-\frac{26}{5}p = -\frac{39}{5}$ $26p = 39$ **[1mark]** Therefore, $p = 1.5$ **[1mark]**

8	Here is a circle with equation $x^2 + y^2 = 52$



AB is a chord with equation $y = x - 2$
Find the length of line AB.
Give your answer in the form $m\sqrt{n}$.

Solution

Substitute $y = x - 2$ into $x^2 + y^2 = 52$

$x^2 + (x-2)^2 = 52$ **[1mark]**

$x^2 + x^2 - 4x + 4 = 52$

$2x^2 - 4x - 48 = 0$

$x^2 - 2x - 24 = 0$ **[1mark]**

$(x+4)(x-6) = 0$

$x = -4$ or $x = 6$ **[1mark]**

When $x = 6$ then, $y = 6 - 2 = 4$

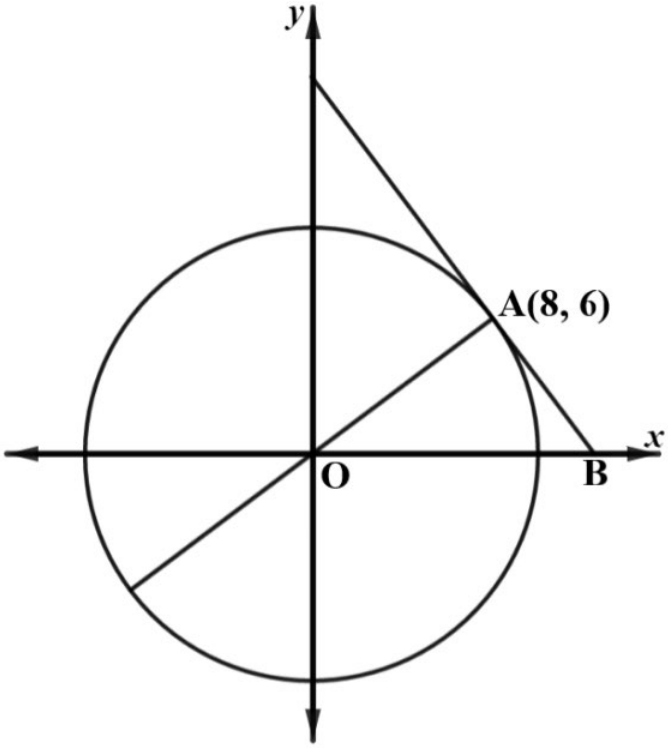
Therefore, B $= (6, 4)$ **[1mark]**

When $x = -4$ then, $y = -4 - 2 = -6$

Therefore, A$= (-4, -6)$ **[1mark]**

$AB = \sqrt{(6 - -4)^2 + (4 - -6)^2} = \sqrt{200} = 10\sqrt{2}$ **[1mark]**

9	Here is a circle with equation $x^2 + y^2 = 100$



A tangent touches the circle at point A (8, 6)
B is the point where the tangent intersects the x-axis
Find the length of AB

Solution

The gradient of the radius, $m = \frac{6-0}{8-0} = \frac{3}{4}$ **[1mark]**

Therefore the gradient of the tangent is $-\frac{4}{3}$ **[1mark]**

The equation of the tangent is $y = -\frac{4}{3}x + c$

Substitute the coordinates of A into $y = -\frac{4}{3}x + c$

$6 = -\frac{4}{3} \times 8 + c$

$6 = -\frac{32}{3} + c$

$c = 6 + \frac{32}{3} = \frac{50}{3}$ **[1mark]**

Hence, the equation is $y = -\frac{4}{3}x + \frac{50}{3}$

Point B is the x-intercept of the tangent

Therefore, $0 = -\frac{4}{3}x + \frac{50}{3}$ **[1mark]**

$\frac{4x}{3} = \frac{50}{3}$

Therefore, $4x = 50$

$x = 12.5$ **[1mark]**

Hence, the coordinates of B are (12.5, 0)

The length of AB is $\sqrt{(12.5-8)^2 + (0-6)^2} = 7.5$ **[1mark]**