



FluidMaths

GCSE Mathematics (Grade 9-1)

Problem Solving
Algebraic Fractions
(Four Operations)
Solutions

**The marks shown are for guidance
purposes only**

**The questions are repeated here for your
convenience**

1	<p>Given that $x\frac{1}{2} + 3x\frac{1}{3} = 20\frac{5}{6}$</p> <p>What is the value of the whole number x?</p> <p style="text-align: center;"><u>Solution</u></p> <p>$x + 3x = 4x$ [1mark]</p> <p>$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ [1mark]</p> <p>Therefore, $x\frac{1}{2} + 3x\frac{1}{3} = 4x\frac{5}{6}$</p> <p>$4x\frac{5}{6} = 20\frac{5}{6}$</p> <p>$4x = 20$ [1mark]</p> <p>Hence, $x = 5$ [1mark]</p>
2	<p>Simplify $4\frac{2x}{3} + 5\frac{x}{4}$ as far as possible</p> <p style="text-align: center;"><u>Solution</u></p> <p>$4 + 5 = 9$</p> <p>$\frac{2x}{3} + \frac{x}{4} = \frac{8x+3x}{12}$ [1mark]</p> <p>$\frac{8x+3x}{12} = \frac{11x}{12}$ [1mark]</p> <p>Therefore, $4\frac{2x}{3} + 5\frac{x}{4} = 9\frac{11x}{12}$ [1mark]</p>
3	<p>Given that $\frac{1}{2} + \frac{x}{y} = 1\frac{2}{3}$</p> <p>What is the value of the expression $x + y$?</p> <p style="text-align: center;"><u>Solution</u></p> <p>$\frac{x}{y} = 1\frac{2}{3} - \frac{1}{2}$</p> <p>$\frac{x}{y} = 1\frac{1}{6}$ [1mark]</p> <p>$\frac{x}{y} = \frac{7}{6}$ [1mark]</p> <p>Therefore, $x = 7$ and $y = 6$</p> <p>Hence, $x + y = 7 + 6 = 13$ [1mark]</p>

4

Given that $\frac{a}{4} + \frac{b}{7} = \frac{25}{28}$

where a and b are different whole numbers

What is the value of the expression $a + b$?

Solution

$$\frac{a}{4} + \frac{b}{7} = \frac{7a}{28} + \frac{4b}{28} = \frac{7a+4b}{28} \quad [1\text{mark}]$$

Therefore, $\frac{7a+4b}{28} = \frac{25}{28}$

This means that $7a + 4b = 25$ [1mark]

Since $4b$ is even, then $7a$ must be odd

$7 \times 3 + 4 \times 1 = 25$. $a = 3$ and $b = 1$ [2marks]

Hence, $a + b = 3 + 1 = 4$ [1mark]

5

Given that $\frac{x}{6} + \frac{y}{5} = \frac{33}{30}$

Where x and y are positive whole numbers

What is the value of the expression $\frac{x}{6} - \frac{y}{5}$?

Solution

$$\frac{x}{6} + \frac{y}{5} = \frac{5x}{30} + \frac{6y}{30} = \frac{5x+6y}{30} \quad [1\text{mark}]$$

$$\frac{5x+6y}{30} = \frac{33}{30}$$

Therefore, $5x + 6y = 33$ [1mark]

{You may use trial and improvement}

$5 \times 3 + 6 \times 3 = 33$

So $x = 3$ and $y = 3$ [1mark]

$$\frac{x}{6} - \frac{y}{5} = \frac{3}{6} - \frac{3}{5}$$

$$\frac{3}{6} - \frac{3}{5} = \frac{15}{30} - \frac{18}{30} = -\frac{3}{30}. \text{ Hence, } \frac{x}{6} - \frac{y}{5} = -\frac{3}{30} = -\frac{1}{10}$$

[1mark]

6 Given that $\frac{x}{2} + \frac{x}{2} = 5$ and $\frac{x}{3} - \frac{y}{3} = 4$
Workout the value of y

Solution

$$\frac{x}{2} + \frac{x}{2} = 5$$

$$x = 5 \quad [1\text{mark}]$$

$$\text{Therefore, } \frac{5}{3} - \frac{y}{3} = 4$$

$$\frac{y}{3} = \frac{5}{3} - 4 \quad [1\text{mark}]$$

$$\frac{y}{3} = -\frac{7}{3}$$

$$\text{Hence, } y = -7 \quad [1\text{mark}]$$

7 Given that $\frac{x}{5} + \frac{6}{7} = \frac{86}{35}$
What is the value of x ?

Solution

$$\frac{x}{5} + \frac{6}{7} = \frac{7x}{35} + \frac{30}{35} \quad [1\text{mark}]$$

$$\text{Therefore, } 7x + 30 = 86 \quad [1\text{mark}]$$

$$7x = 56$$

$$\text{Hence, } x = 8 \quad [1\text{mark}]$$

8 Given that $\frac{x}{8} \times \frac{5}{x} = \frac{px}{3} \div \frac{4}{5}$

Find the value of p when $x = 4$?

Solution

$$\frac{x}{8} \times \frac{5}{x} = \frac{px}{3} \div \frac{4}{5} \quad \{\text{Cancel the common factors}\}$$

$$\frac{5}{8} = \frac{4p}{3} \times \frac{5}{4} \quad \mathbf{[1\text{mark}]}$$

$$\frac{5}{8} = \frac{5p}{3} \quad \{\text{Cross multiply}\} \quad \mathbf{[1\text{mark}]}$$

$$15 = 40p$$

$$p = \frac{15}{40}$$

$$\text{Hence, } p = \frac{3}{5} \quad \mathbf{[1\text{mark}]}$$

9 Simplify fully $\frac{x^2+x-6}{x^2-3x+2} \times \frac{x^2-x}{x^2-9}$

Solution

$$\frac{x^2+x-6}{x^2-3x+2} \times \frac{x^2-x}{x^2-9} \quad \{\text{Factorise}\}$$

$$\frac{(x-2)(x+3)}{(x-2)(x-1)} \times \frac{x(x-1)}{(x+3)(x-3)} \quad \mathbf{[2\text{marks}]}$$

{Cancel out the common factors}

$$\text{Therefore, } \frac{x^2+x-6}{x^2-3x+2} \times \frac{x^2-x}{x^2-9} = \frac{x}{x-3} \quad \mathbf{[1\text{mark}]}$$

10 Given that $\frac{(x+4)^2}{3x} \div \frac{2x+8}{15x} = ax + 10$,
What is the value of a as a decimal?

Solution

$$\frac{(x+4)^2}{3x} \div \frac{2x+8}{15x} = \frac{(x+4)(x+4)}{3x} \times \frac{15x}{2(x+4)} \quad [1\text{mark}]$$

$$= \frac{5(x+4)}{2} = \frac{5x+20}{2} \quad [1\text{mark}]$$

$$= \frac{5x}{2} + \frac{20}{2}$$

$$= 2.5x + 10 \quad [1\text{mark}]$$

$$\text{Hence, } a = 2.5 \quad [1\text{mark}]$$

11 Given that $\frac{x-10}{x+2} \div \frac{1}{(x^2-4)} = (x+a)^2 + b$

What is the value of the expression $a - b$?

Solution

$$\frac{x-10}{x+2} \div \frac{1}{(x^2-4)} = \frac{x-10}{x+2} \times \frac{(x+2)(x-2)}{1} \quad [1\text{mark}]$$

$$= \frac{(x-10)(x-2)}{1} \quad [1\text{mark}]$$

$$= x^2 - 12x + 20 \quad \{\text{Write in complete square form}\}$$

$$x^2 - 12x + 20 = (x - 6)^2 - 36 + 20$$

$$= (x - 6)^2 - 16 \quad [1\text{mark}]$$

Therefore, $a = -6$ and $b = -16$

$$\text{Hence, } a - b = -6 - -16 = 10 \quad [1\text{mark}]$$

12 Given that $\frac{x^2+9x+14}{x^2+10x+21} \div \frac{x^2+10x+16}{x^2+2x-3} = \frac{x+a}{x+b}$
 What is the value of the expression $(a + b)^2$

Solution

$$\frac{x^2+9x+14}{x^2+10x+21} \div \frac{x^2+10x+16}{x^2+2x-3} \quad \{\text{Factorise}\}$$

$$= \frac{(x+7)(x+2)}{(x+7)(x+3)} \div \frac{(x+2)(x+8)}{(x-1)(x+3)} \quad \mathbf{[2marks]}$$

$$= \frac{(x+7)(x+2)}{(x+7)(x+3)} \times \frac{(x-1)(x+3)}{(x+2)(x+8)} = \frac{x-1}{x+8} \quad \mathbf{[1mark]}$$

Therefore, $a = -1$ and $b = 8$ $\mathbf{[1mark]}$

Hence, $(a + b)^2 = (-1 + 8)^2 = 49$ $\mathbf{[1mark]}$

13 Given that $\frac{2x^2+5x+2}{x^2-4} \div \frac{2x^2+x}{x^2+x-6} = \frac{(x+k)}{x}$

What is the value of k ?

Solution

$$\frac{2x^2+5x+2}{x^2-4} \div \frac{2x^2+x}{x^2+x-6} = \frac{(2x+1)(x+2)}{(x+2)(x-2)} \div \frac{x(2x+1)}{(x+3)(x-2)}$$

$\mathbf{[2marks]}$

$$= \frac{(2x+1)(x+2)}{(x+2)(x-2)} \times \frac{(x+3)(x-2)}{x(2x+1)} \quad \mathbf{[1mark]}$$

$$= \frac{x+3}{x} \quad \mathbf{[1mark]}$$

Hence, $k = 3$ $\mathbf{[1mark]}$

14 Given that $\frac{2x^2-7x-15}{x^2-25} \times \frac{x^2-2x-35}{x^2-49} = \frac{ax+b}{cx+d}$
 What is the value of the expression $(ab + cd)$?

Solution

$$\frac{2x^2-7x-15}{x^2-25} \times \frac{x^2-2x-35}{x^2-49} = \frac{(2x+3)(x-5)}{(x-5)(x+5)} \times \frac{(x+5)(x-7)}{(x-7)(x+7)}$$

[2marks]

$$= \frac{2x+3}{x+7} \quad [1\text{mark}]$$

Therefore, $a = 2$; $b = 3$; $c = 1$ and $d = 7$ [1mark]

Hence, $ab + cd = 2 \times 3 + 1 \times 7$

$$= 6 + 7 = 13 \quad [1\text{mark}]$$

15 Show that $\frac{(x-3)(5x^2+32x-21)}{25x^2-9} \div \frac{x+7}{10x^2+11x+3}$

can be written in the form $ax^2 + bx + c$

where a , b and c are integers

Solution

$$\frac{(x-3)(5x^2+32x-21)}{25x^2-9} \div \frac{x+7}{10x^2+11x+3}$$

$$= \frac{(x-3)(5x-3)(x+7)}{(5x-3)(5x+3)} \times \frac{(5x+3)(2x+1)}{x+7} \quad [4\text{marks}]$$

$$= (x-3)(2x+1) = 2x^2 - 5x - 3 \quad [1\text{mark}]$$