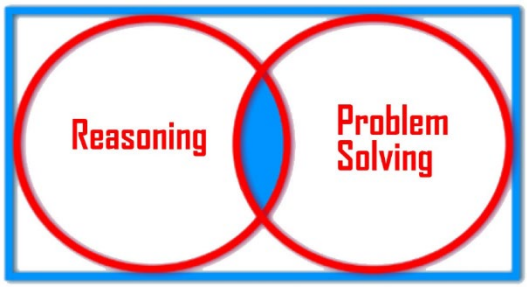
GCSE Mathematics (Grade 9-1)

Problem Solving – Sample 4 (Solutions)



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{Aimed at students working towards a Grade 7 or 6}

The Grades and Marks shown are for guidance purposes only

Questions are repeated here for your convenience

1 A bag contains 6 Red, 5 Yellow and some White counters

The probability of picking a Red counter from the bag is $\frac{3}{10}$

Show that the probability of picking a White counter is $\frac{9}{20}$

Solution

Let the number of white counters in the bag be w

Then, the total number of counters in the bag will be

$6 + 5 + w = 11 + w$

[1mark]

The probability of picking a Red counter from the bag will be $= \frac{6}{11+w}$

Therefore, $\frac{6}{11+w} = \frac{3}{10}$ **[1mark]**

So we have $60 = 33 + 3w$ {Subtract 33 from both sides}

$27 = 3w$ **[1mark]**

Therefore, $w = 9$ So, there are 9 white counters in the bag

The total number of counters is 6 + 5 + 9 = 20 **[1mark]**

Hence, the probability of picking a white counter $= \frac{9}{20}$ **[1mark]**

2 Keira, Logan and Lewis share £900 in the ratio $x : 7 : x + 5$

Lewis received £400. Show that $x = 3$

Solution

The total ratio = $x + 7 + x + 5 = 2x + 12$ **[1mark]**

The fraction of the total amount Lewis received will be given as

$\frac{x+5}{2x+12} \times 900 = 400$ **[1mark]**

We can now solve the above equation to find the value of x.

$\frac{x+5}{2x+12} \times 900 = 400$ {Divide both sides by 900}

$\frac{x+5}{2x+12} = \frac{400}{900}$

That is $\frac{x+5}{2x+12} = \frac{4}{9}$ {Cross multiply} **[1mark]**

$9x + 45 = 8x + 48$

$x = 3$ **[1mark]**

{Alternatively, you could replace x with 3 in the given ratio and calculate Lewis's share to check if it will be £400}

3 Given that $A = 10x^{\frac{1}{2}} - 6\left(x^{\frac{1}{2}} + 3y^3\right)$.

Make x the subject of the formula

Solution

Expand the brackets

$A = 10x^{\frac{1}{2}} - 6(x^{\frac{1}{2}} + 3y^3)$

$A = 10x^{\frac{1}{2}} - 6x^{\frac{1}{2}} - 18y^3$ {Add $18y^3$ to both sides} **[1mark]**

$A + 18y^3 = 4x^{\frac{1}{2}}$ {Notice that $10x^{\frac{1}{2}}$ and $6x^{\frac{1}{2}}$ are like terms} **[1mark]**

Now divide both sides by 4

$\frac{A+18y^3}{4} = x^{\frac{1}{2}}$ {Square both sides} **[1mark]**

Hence $x = \left(\frac{A+18y^3}{4}\right)^2$ **[1mark]**

4 Given that, $x^2 + 8x - 8 = (x + 2p)^2 + 4pq$

Find the exact values of p and q

Solution

Write the expression on the LHS in completed square form

$x^2 + 8x - 8 = (x + 4)^2 - 16 - 8$ {Simplify further} **[1mark]**

$x^2 + 8x - 8 = (x + 4)^2 - 24$

Therefore, $(x + 4)^2 - 24 = (x + 2p)^2 + 4pq$ **[1mark]**

Now compare the coefficients

We find that, $2p = 4$

Therefore, $p = 2$ **[1mark]**

Also $4pq = -24$ {Divide both sides by 4}

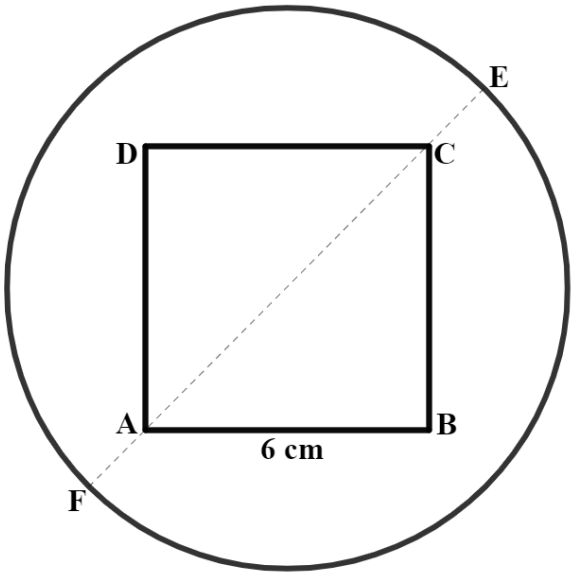
So, $pq = -6$ **[1mark]**

Sub in the value of $p = 2$

$2q = -6$

Therefore, $q = -3$ **[1mark]**

5 ABCD is a square of side 6 cm drawn inside a circle with diameter EF



The area of the circle is 72π cm^2
Calculate the distance CE. Give your answer as an exact value.

Solution

{Area of a circle $= \pi r^2$}
Therefore, $\pi r^2 = 72\pi$
$r = \sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$ **[1mark]**
Therefore, the diameter of the circle will be $12\sqrt{2}$ **[1mark]**

Now calculate the diagonal of the square using Pythagoras theorem
$d^2 = 6^2 + 6^2$ {Where d is the diagonal of the square}
$d^2 = 72$
$d = \sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$ **[1mark]**

Therefore, the difference between the length of the diagonal of the square and the diameter of the circle will be
$12\sqrt{6} - 6\sqrt{2} = 6\sqrt{2}$ **[1mark]**

Therefore, CE $= \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ **[1mark]**