GCSE Mathematics (Grade 9-1)

Problem Solving – (Sample 1) Solutions

1. Given that, $5 : x(x+4) = 3 : 7$
   Show that $3x^2 + 12x - 35 = 0$

Solution

Write the ratio as a proportional equation

$\frac{5}{x(x+4)} = \frac{3}{7}$ {Cross multiply} **[1mark]**

$3x(x+4) = 35$ {Expand the brackets}

$3x^2 + 12x = 35$ {Subtract 35 from both sides} **[1mark]**

Hence, $3x^2 + 12x - 35 = 0$ **[1mark]**

2. If $\left(\sqrt{p} + \sqrt{5p}\right)^2 = 10 + q\sqrt{5}$, Show that $q = 3\frac{1}{3}$

Solution

Expand the brackets on the LHS

$\left(\sqrt{p} + \sqrt{5p}\right)^2 = \left(\sqrt{p} + \sqrt{5p}\right)\left(\sqrt{p} + \sqrt{5p}\right) = p + p\sqrt{5} + p\sqrt{5} + 5p$

{Simplify by collecting the like terms} **[1mark]**

$p + p\sqrt{5} + p\sqrt{5} + 5p = 6p + 2p\sqrt{5}$ **[1mark]**

{Now compare this expression with the expression on the RHS}

We notice that $6p = 10$

Therefore, $p = \frac{10}{6} = \frac{5}{3}$ **[1mark]**

Also, $q = 2p$

Therefore, $q = 2 \times \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}$ **[1mark]**

3. The roots of a quadratic equation y = f(x) are given as $x = \frac{-7\pm\sqrt{49-24}}{6}$

The coordinate $(3, k)$ lies on f(x).

Find the exact value of k

Solution

We need to find the quadratic equation with such roots

If $ax^2 + bx + c = 0$ then, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Now compare the quadratic formula above with the given value of x in the question: $x = \frac{-7\pm\sqrt{49-24}}{6}$

Notice that

$-b = -7$

Therefore, $b = 7$

$2a = 6$

Therefore, $a = 3$

$4ac = 24$ Substitute a = 3 into this equation to find the value of c

$4 \times 3 \times c = 24$

Therefore, $c = 2$ **[2marks]**

Hence, the quadratic equation is $3x^2 + 7x + 2 = 0$ **[1mark]**

Now, to find the value of k, we need to substitute x = 3 into the above equation

If x = 3 then, $3(3)^2 + 7(3) + 2 = 0$ **[1mark]**

$27 + 21 + 2 = 50$

Therefore, $k = 50$ **[1mark]**

4. Given that $10x^2 - 9xy + 2y^2 = 0$,
Find the possible values of x in terms of y

Solution

We need to factorise the equation into double brackets

We will then have to solve the factors in terms of x and y

$10x^2 - 9xy + 2y^2 = 0$

Factorise into a double bracket as follows:

$10 \times 2 = 20$ {Multiplying the coefficients of x^2 and y^2}

We now need to find the factors of +20 which we can add to get -9

Therefore, both factors must be negative

The required factors will be -5 and -4

Therefore, $10x^2 - 9xy + 2y^2 = (2x - y)(5x - 2y)$ **[2marks]**

Hence, $(2x - y)(5x - 2y) = 0$

Now solve each factor separately

Either $2x - y = 0$

Therefore, $2x = y$ {Divide both sides by y and by 2}

So we have $x = \frac{1}{2}y$ **[1mark]**

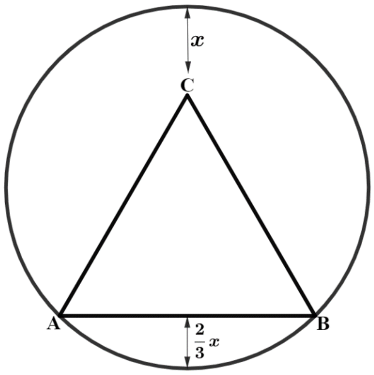
or

$5x - 2y = 0$

$5x = 2y$ {Divide both sides by y and by 5}

$x = \frac{2}{5}y$ **[1mark]**

5. ABC is equilateral triangle of side 6 cm which is drawn inside a circle.
The area of the circle is 108π cm^2



The vertex C lies on the diameter of the circle. Calculate the exact value of x

Solution

calculate x by finding the difference between the length of the diameter of the circle and the height of the triangle

$\sin 60 = \frac{h}{6}$ where h is the height of the triangle.

{Note that each angle in an equilateral triangle is 60°}

$h = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$ {Note that exact value of $\sin 60 = \frac{\sqrt{3}}{2}$} **[1mark]**

{Area of a circle = πr^2} Therefore, $108\pi = \pi r^2$

$r^2 = 108$ Therefore, $r = \sqrt{108} = 6\sqrt{3}$

Hence, the length of the diameter will be $12\sqrt{3}$ **[1mark]**

Difference between the diameter of the circle and height of the triangle

$12\sqrt{3} - 3\sqrt{3} = 9\sqrt{3}$ **[1mark]**

Therefore, $x + \frac{2}{3}x = 9\sqrt{3}$ **[1mark]**

$\frac{5}{3}x = 9\sqrt{3}$

Therefore, $5x = 27\sqrt{3}$. Hence, $x = \frac{27\sqrt{3}}{5}$ **[1mark]**