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Year 1 – AS

Quadratics (Sample Questions)
Solutions

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 A quadratic function is defined on a set of real numbers as

$$f(x) = x^2 + bx + c$$

The roots of f(x) are given as $x = 3\sqrt{2}$ and $x = -\sqrt{2}$

Find the minimum point of the function

Solution

If $x = 3\sqrt{2}$ and $x = -\sqrt{2}$ are the roots for f(x),

then $f(x) = (x - 3\sqrt{2})(x + \sqrt{2})$ **[1mark]**

$f(x) = x^2 - \left(2\sqrt{2}\right)x - 6$

[1mark]

$f(x) = \left(x - \sqrt{2}\right)^2 - 2 - 6$

$f(x) = \left(x - \sqrt{2}\right)^2 - 8$ **[1mark]**

Hence, the minimum point for f(x) is $(\sqrt{2}, -8)$ **[1mark]**

2	Given that $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{8}{27}\right)^{-\frac{2}{3}+p}$ Find the possible values of p Give your answers as exact values **<u>Solution</u>** Simplify the fraction inside the brackets on the RHS $\left(\frac{8}{27}\right)^{-\frac{2}{3}+p} = \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}+p}$ **[1mark]** $\left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}} = \left(\frac{2}{3}\right)^{3\times\left(-\frac{2}{3}+p\right)} = \left(\frac{2}{3}\right)^{-2+3p}$ **[1mark]** Therefore, $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{2}{3}\right)^{-2+3p}$ Notice that the fractions in the brackets do not much, therefore, we need to apply the negative index law: $\{\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}\}$ Therefore, $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{3}{2}\right)^{2-3p}$ **[1mark]** Hence, $p^2 = 2 - 3p$ $p^2 + 3p - 2 = 0$ **[1mark]** Solve using the quadratic solver in your calculator Therefore, $p = \frac{-3+\sqrt{17}}{2}$ or $p = \frac{-3-\sqrt{17}}{2}$ **[1mark]**

3	Two functions f and g are defined on a set of real numbers as $f(x) = x^2 + Px + 7$ and $g(x) = -2x^2 - 6x + 1$ The functions intersect at points $A(m, n)$ and $B(-3, 1)$ Find: a) The value of P b) The values of m and n c) Hence find the minimum point of $f(x)$ **Solution** a) To find the value of P, we need to substitute the given coordinate of intersection into $f(x)$ When $x = -3, y = 1$ Therefore, we have, $1 = (-3)^2 + (-3)P + 7$ **[1mark]** $1 = -3P + 16$ $-3P = -15$ Hence $P = 5$ **[1mark]** Therefore, $f(x) = x^2 + 5x + 7$ b) Now equate the two functions and solve to find the other point of intersection $x^2 + 5x + 7 = -2x^2 - 6x + 1$ **[1mark]** $3x^2 + 11x + 6 = 0$ $(x + 3)(3x + 2) = 0$ Therefore, $x = -3$ or $x = -\frac{2}{3}$ **[1mark]** So, $m = -\frac{2}{3}$

Substitute $m = -\frac{2}{3}$ into f(x) to find n

$f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^2 + 5\left(-\frac{2}{3}\right) + 7 = \frac{37}{9}$ **[1mark]**

Therefore, $n = \frac{37}{9}$

c)Write f(x) in completed square form

$f(x) = x^2 + 5x + 7$

$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + 7$ **[1mark]**

$= \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}$

Hence, the minimum point for f(x) is $\left(-\frac{5}{2}, \frac{3}{4}\right)$ **[1mark]**

4	A function is defined as $f(x) = x^2 + 6x + k$ When $f(x)$ is divided by $(x - a)$, the remainder is the same as when $f(x)$ is divided by $(x - 2a)$ a) Find the value of a such that $a < 0$ b) The remainder when $f(x)$ is divided by $(x + 3a)$ is 5 Calculate the value of k **<u>Solution</u>** a) If $(x - a)$ leaves a remainder, then the remainder will be $f(a) = (a)^2 + 6(a) + k$ $= a^2 + 6a + k$ **[1mark]** Also, if $(x - 2a)$ leaves a remainder, then the remainder will be $f(2a) = (2a)^2 + 6(2a) + k$ $= 4a^2 + 12a + k$ **[1mark]** The remainders are the same Therefore, $4a^2 + 12a + k = a^2 + 6a + k$ **[1mark]** $3a^2 + 6a = 0$ $a^2 + 2a = 0$ So, $a = 0$ or $a = -2$ **[1mark]** Therefore, $a = -2$

b) If $(x+3a)$ leaves a remainder, then the remainder will be $\mathrm{f}(-3a) = (-3a)^2 + 6(-3a) + k$

$= 9a^2 - 18a + k$

Therefore, $9a^2 - 18a + k = 5$ **[1mark]**

Substitute $a = -2$ into $9a^2 - 18a + k = 5$

We have $9(-2)^2 - 18(-2) + k = 5$

Therefore, $k = 67$ **[1mark]**

5	A function f is defined on a set of real numbers as $$f(x) = C - 2x - 4x^2$$ where C is a constant a) Given that the maximum value of $f(x)$ is $\frac{21}{4}$ find the value of C b) Hence determine whether there exists any point of contact between the line $y = 2x + 6$ and $f(x)$ and find the coordinates of such a point if it exists. **<u>Solution</u>** a) Write the function in completed square form $f(x) = -4x^2 - 2x + C$ $= -4\left(x^2 + \frac{1}{2}x - \frac{C}{4}\right)$ **[1mark]** $= -4\left[\left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{C}{4}\right]$ **[1mark]** $= -4\left[\left(x + \frac{1}{4}\right)^2 - \frac{(4C+1)}{16}\right]$ $= -4\left(x + \frac{1}{4}\right)^2 + \frac{(4C+1)}{4}$ **[1mark]** Therefore, $\frac{21}{4} = \frac{4C+1}{4}$ **[1mark]** So $4C + 1 = 21$ $4C = 20$ Therefore, $C = 5$ **[1mark]** Hence, $f(x) = -4x^2 - 2x + 5$

	b) Make the two equations equal $-4x^2 - 2x + 5 = 2x + 6$ **[1mark]** $-4x^2 - 4x - 1 = 0$ **[1mark]** If $y = 2x + 6$ is a tangent to $\mathrm{f}(x) = -4x^2 - 2x + 5$ then $b^2 - 4ac = 0$ **[1mark]** $a = -4$ $b = -4$ $c = -1$ We have; $(-4)^2 - 4(-4)(-1) = 0$ Therefore, $y = 2x + 6$ is a tangent to $\mathrm{f}(x) = -4x^2 - 2x + 5$ **[1mark]** Now we can find the point of contact by solving $-4x^2 - 4x - 1 = 0$ $4x^2 + 4x + 1 = 0$ Factorise into $(2x + 1)^2 = 0$ therefore, $x = -\frac{1}{2}$ **[1mark]** If $x = -\frac{1}{2}$ then $y = 2\left(-\frac{1}{2}\right) + 6 = 5$ Hence, the point of contact is the coordinate $\left(-\frac{1}{2}, 5\right)$ **[1mark]**