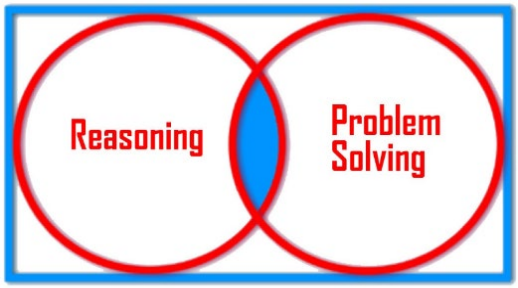

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Year 1 – AS

Exponentials and Logarithms (Sample Questions) (Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 Given that $\log x = a$ and $\log y = b$

Express $\log\left(\frac{10x^3}{\sqrt{y}}\right)$ in terms of a and b

Solution

$\log\frac{10a^3}{\sqrt{y}} = \log 10x^3 - \log\sqrt{y}$ **[1mark]**

$= \log_{10} 10 + \log_{10} x^3 - \log_{10} y^{\frac{1}{2}}$ **[1mark]**

$= 1 + 3\log_{10} x + \frac{1}{2}\log_{10} y$ **[1mark]**

Substitute a for $\log x$ and b for $\log y$

Therefore, we have $1 + 3a + \frac{1}{2}b$ **[1mark]**

2 Given that $y = 5 + 2^{5x+1}$

Find x in terms of y

Solution

$y = 5 + 2^{5x+1}$

$y - 5 = 2^{5x+1}$

$\log(y-5) = \log 2^{5x+1}$ **[1mark]**

$\log(y-5) = (5x+1)\log 2$ **[1mark]**

$\log(y-5) = 5x\log 2 + \log 2$ **[1mark]**

$\log(y-5) - \log 2 = 5x\log 2$ **[1mark]**

Hence, $x = \frac{\log(y-5) - \log 2}{5\log 2}$ **[1mark]**

3 Given that $\log x = (p+q)$ and $\log y = (p-q)$

Express

a) $\log x^3y^3$ in terms of p

b) $\log\left(\frac{x}{100y}\right)$ in terms of q

Solution

a) $\log x^3y^3 = \log x^3 + \log y^3$ **[1mark]**

$= 3\log x + 3\log y$

$= 3(p+q) + 3(p-q)$ **[1mark]**

$= 3p + 3q + 3p - 3q$ **[1mark]**

Therefore, $\log x^3y^3 = 6p$ **[1mark]**

b) $\log\left(\frac{x}{100y}\right) = \log x - \log 100y$ **[1mark]**

$= \log x - (\log 100 + \log y)$

$= \log x - (\log 10^2 + \log y)$ **[1mark]**

$= \log x - (2\log 10 + \log y)$ {Note that $\log_{10} 10 = 1$}

$= \log x - 2 - \log y$ **[1mark]**

Substitute the values of $\log x$ and $\log y$

So, we have $\log\left(\frac{x}{100y}\right) = (p+q) - 2 - (p-q)$

$= p + q - 2 - p + q$ **[1mark]**

Hence, $\log\left(\frac{x}{100y}\right) = 2q - 2$ **[1mark]**

4 Given that $\log 2a + 2\log(b+1) = \log(a+1)$,

a) Express b in terms of a

b) Determine the possible values of b when $a = 8$

Solution

a) $\log 2a + 2\log(b+1) = \log(a+1)$

$2\log(b+1) = \log(a+1) - \log 2a$ **[1mark]**

$\log(b+1)^2 = \log\left(\frac{a+1}{2a}\right)$ **[1mark]**

Therefore, $(b+1)^2 = \frac{a+1}{2a}$ **[1mark]**

$b + 1 = \sqrt{\frac{a+1}{2a}}$ **[1mark]**

Hence, $b = \sqrt{\frac{a+1}{2a}} - 1$ **[1mark]**

b) $b = \sqrt{\frac{8+1}{2\times 8}} - 1$

$= \sqrt{\frac{9}{16}} - 1$

Therefore, $b = \frac{3}{4} - 1 = -\frac{1}{4}$ **[1mark]**

or $b = -\frac{3}{4} - 1 = -\frac{7}{4}$ **[1mark]**

5 The pressure P and the volume V of a gas are related by the formula $P = kV^n$ where k and n are constants
The information in the table below was obtained in an experiment in which P and V were measured

V	200	150	100	80	50
P	76	115	200	280	530

a)Express $P = kV^n$ in the form $y = mx + c$
b)Find the values of k and n

Solution

a)$P = kV^n$

$\log P = \log(kV^n)$ **[1mark]**

$\log P = \log k + n \log V$

Hence, $\log P = n \log V + \log k$ **[1mark]**

b)Use corresponding values of V and P from the table

When $P = 76; V = 200$
Therefore, $76 = k(200^n)$--Equation 1 **[1mark]**

When $P = 115; V = 150$
Therefore, $115 = k(150^n)$--Equation 2 **[1mark]**

Divide Equation 1 by Equation 2

$$\frac{76}{115} = \frac{k(200)^n}{k(150)^n}$$

$$\frac{76}{115} = \left(\frac{4}{3}\right)^n$$

$$\log\left(\frac{76}{115}\right) = n\log\left(\frac{4}{3}\right)$$

Therefore, $n = \frac{\log\left(\frac{76}{115}\right)}{\log\left(\frac{4}{3}\right)} = -1.44(3\text{sf})$ **[1mark]**

Now substitute $n = -1.44$ into Equation 1

$$76 = k(200^{-1.44})$$

$$k = \frac{76}{200^{-1.44}} = 15600\ (3\text{sf})$$ **[1mark]**

Hence, $\mathrm{P} = 15600(\mathrm{V})^{-1.44}$