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**Year 2 (A-Level)**

**Integration (sample questions solutions)**

**The marks shown are for guidance purposes only**

**The questions are repeated here for your convenience**

**1** Given that

$$\int_k^{3k} 3x dx = 48$$

Calculate the possible values of  $k$

**Solution**

$$\int_k^{3k} 3x dx = 48$$

$$\left[ \frac{3}{2} x^2 \right]_k^{3k} = 48$$

**[2Marks]**

$$\left[ \frac{3}{2} (3k)^2 \right] - \left[ \frac{3}{2} (k)^2 \right] = 48$$

$$\frac{27}{2} k^2 - \frac{3}{2} k^2 = 48$$

**[1Mark]**

$$12k^2 = 48$$

$$k^2 = 4$$

$$k = \pm 2$$

**[1Mark]**

2 A function is defined on a set of real numbers as

$$f(x) = 10x^2 - kx$$

Given that

$$\int_1^3 f(x) dx = \frac{116}{3}$$

a) Find the value of  $k$

b) Hence evaluate  $f(-6)$

**Solution**

a)

$$\int_1^3 (10x^2 - kx) dx = \frac{116}{3}$$

$$\left[ \frac{10x^3}{3} - \frac{kx^2}{2} \right]_1^3 = \frac{116}{3}$$

**[2marks]**

$$\left[ \frac{10(3)^3}{3} - \frac{k(3)^2}{2} \right] - \left[ \frac{10}{3} - \frac{k}{2} \right] = \frac{116}{3}$$

**[1mark]**

$$90 - \frac{9k}{2} - \frac{10}{3} + \frac{k}{2} = \frac{116}{3}$$

**[1mark]**

$$\frac{-9k+k}{2} = \frac{116}{3} + \frac{10}{3} - 90$$

$$\frac{-8k}{2} = -48$$

$$-4k = -48$$

$$k = 12$$

**[1mark]**

Hence,  $f(x) = 10x^2 - 12x$

b)  $f(-6) = 10x^2 - 12x$

$$= 10(-6)^2 - 12(-6)$$

$$= 432$$

**[1mark]**

**3** A function is defined on a set of real numbers as

$$f(x) = x^2 - 4x$$

a) Illustrate

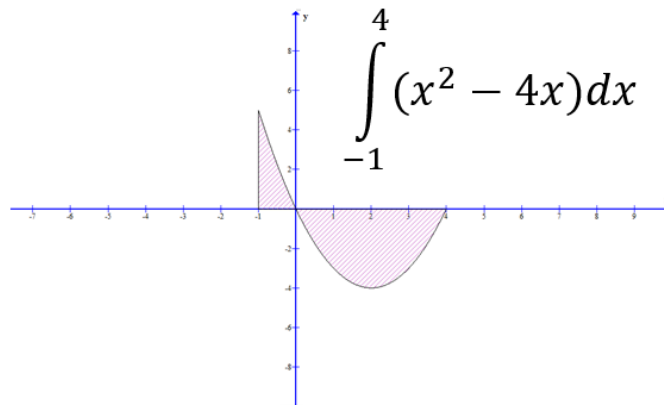
$$\int_{-1}^4 (x^2 - 4x) dx$$

on a graph of  $f(x)$

b) Hence evaluate  $\int_{-1}^4 f(x) dx$

**Solution**

a) This is the area between  $-1$  and  $4$  on the graph  
**[3marks]**



b)

$$\int_{-1}^4 (x^2 - 4x) dx = \left[ \frac{x^3}{3} - \frac{4x^2}{2} \right]_{-1}^4 \quad \text{[2marks]}$$

$$\int_{-1}^4 (x^2 - 4x) dx = \left[ \frac{(4)^3}{3} - \frac{4(4)^2}{2} \right] - \left[ \frac{(-1)^3}{3} - \frac{4(-1)^2}{2} \right]$$

$$= \frac{64}{3} - 32 + \frac{1}{3} + 2 \quad \text{[1mark]}$$

$$\int_{-1}^4 (x^2 - 4x) dx = -\frac{25}{3} \quad \text{[1mark]}$$

4 The functions  $y = x^3 - 2x^2 - x + 2$  and  $y = x^2 - 1$  intersect at points A, B and  $(-1, 0)$

a) Find the coordinates of A and B

b) Show that the area between A and B is a whole number

**Solution**

a) Set the two equations equal and solve to find  $x$

$$x^3 - 2x^2 - x + 2 = x^2 - 1$$

$$x^3 - 3x^2 - x + 3 = 0$$

$$x^2(x - 3) - 1(x - 3) = 0$$

$$(x - 3)(x^2 - 1) = 0$$

Therefore,  $x = 3, \pm 1$      **[2marks]**

When  $x = 3$

$$y = (3)^2 - 1 = 8$$

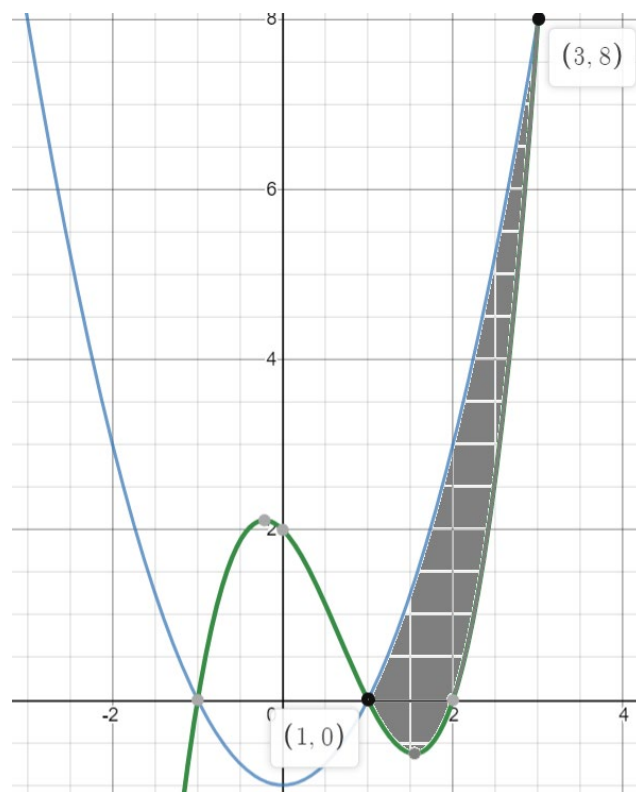
Therefore, A = (3, 8)     **[1mark]**

When  $x = 1$

$$y = (1)^2 - 1 = 0$$

Therefore, B = (1, 0)     **[1mark]**

b) A sketch will help you visualise the required area



$$\begin{aligned} & \int_1^3 (x^2 - 1) dx - \int_1^3 (x^3 - 2x^2 - x + 2) dx \\ &= \left[ \frac{x^3}{3} - x \right]_1^3 - \left[ \frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x \right]_1^3 \\ &= \left[ 6 - -\frac{2}{3} \right] - \left[ \frac{15}{4} - \frac{13}{12} \right] \\ &= \left[ \frac{20}{3} - \frac{8}{3} \right] = 4 \end{aligned}$$

The area between A and B is 4square units **[4marks]**

5 Given that,  $u = 2x^3 + 3$  evaluate the integral below

$$\int_{-1}^{\frac{1}{2}} x^2(2x^3 + 3)^4 dx$$

**Solution**

Since  $u = 2x^3 + 3$

Then,  $\frac{du}{dx} = 6x^2$  [1Mark]

Therefore,  $du = 6x^2 dx$

$$x^2 dx = \frac{1}{6} du$$
 [1Mark]

At  $x = -1$   $u = -2 + 3 = 1$  [1Mark]

At  $x = \frac{1}{2}$   $u = \frac{2}{8} + 3 = \frac{13}{4}$  [1Mark]

Therefore,

$$\int_{-1}^{\frac{1}{2}} x^2(2x^3 + 3)^4 dx = \int_1^{\frac{13}{4}} \left(\frac{1}{6}\right) u^4 du$$

$$= \frac{1}{6} \left[ \frac{u^5}{5} \right]_1^{\frac{13}{4}}$$
 [1mark]

$$= \frac{1}{6} \left[ \frac{\left(\frac{13}{4}\right)^5}{5} \right] - \frac{1}{6} \left[ \frac{1}{5} \right]$$
 [1mark]

$$= \frac{1}{6} \left( \frac{371293}{1024} \times \frac{1}{5} \right) - \frac{1}{30}$$

$$\frac{371293}{30720} - \frac{1}{30} = 12.1(3sf)$$
 [1mark]