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Year 1 (AS)

Binomial Expansion – (Sample Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1	Express $\left(\sqrt{5}+2\sqrt{7}\right)^5$ in the form $p\sqrt{5}+q\sqrt{7}$ where *p* and *q* are whole numbers to be found **<u>Solution</u>** $\left(\sqrt{5}+2\sqrt{7}\right)^5 = \binom{5}{0}\left(\sqrt{5}\right)^5 + \binom{5}{1}\left(\sqrt{5}\right)^4 2\sqrt{7}$ $+\binom{5}{2}\left(\sqrt{5}\right)^3\left(2\sqrt{7}\right)^2 + \binom{5}{3}\left(\sqrt{5}\right)^2\left(2\sqrt{7}\right)^3$ $+\binom{5}{4}\left(\sqrt{5}\right)\left(2\sqrt{7}\right)^4 + \binom{5}{5}\left(2\sqrt{7}\right)^5$ **[2marks]** $= \left(25\sqrt{5}\right) + \left(250\sqrt{7}\right) + \left(1400\sqrt{5}\right) + \left(2800\sqrt{7}\right)$ $+ \left(3920\sqrt{5}\right) + \left(1568\sqrt{7}\right)$ Therefore, $\left(\sqrt{5}+2\sqrt{7}\right)^5 = \left(5345\sqrt{5}\right) + \left(4918\sqrt{7}\right)$ Where $p = 5345$ and $q = 4918$ **[2marks]**

2 Show that ${}^{n}C_3 - {}^{n}C_2 = \frac{1}{6}(n^3 - 6n^3 + 5n)$

Solution

Apply ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

Therefore,

$${}^{n}C_3 - {}^{n}C_2 = \frac{n!}{3!(n-3)!} - \frac{n!}{2!(n-2)!}$$

$$= \frac{n(n-1)(n-2)(n-3)!}{3!(n-3)!} - \frac{n(n-1)(n-2)!}{2!(n-2)!}$$ **[1mark]**

$$= \frac{n(n-1)(n-2)}{3\times 2} - \frac{n(n-1)}{2}$$

$$= \frac{n^3-3n^2+2n}{6} - \frac{3n^2-3n}{6}$$ **[1mark]**

$$= \frac{n^3-6n^3+5n}{6}$$ **[1mark]**

$$= \frac{1}{6}(n^3 - 6n^3 + 5n)$$ **[1mark]**

3	The coefficient of x^3 in the expansion of the expression $(5+px)^7$ is 7503125. Find the value of p

Solution

The coefficient of x^3 is 7503125

Therefore, $\binom{7}{3} \times 5^4 \times p^3 = 7503125$ **[1mark]**

$21875p^3 = 7503125$

$p^3 = \frac{7503125}{21875}$ **[1mark]**

$p^3 = 343$

$p = (343)^{\frac{1}{3}}$

$p = 7$ **[1mark]**

4	Find the coefficient of the term in x^5 in the expansion of the expression $(2+5x)(1+2x)^8$

Solution

General term for $(1+2x)^8$ is ${}^{n}C_r = \binom{8}{r}(1)^{8-r}(2x)^{r}$

The term containing x^4 can be found as follows

$\binom{8}{4} \times 1^4 \times (2x)^4$

$70 \times 2^4 \times x^4$

$1120 \times x^4$ **[1mark]**

When this is multiplied by $5x$ from the first bracket, we will obtain a term in x^5

Also, the term containing x^5 can be found as follows

$\binom{8}{5} \times 1^3 \times (2x)^5$

$56 \times 2^5 \times x^5$

$1792x^5$ **[1mark]**

When this is multiplied by 2 from the first bracket, we will obtain another term in x^5

Therefore for the terms in x^5, we will have

$= (2+5x)(1120x^4 + 1792x^5)$

$= 2 \times 1792x^5 + 5 \times 1120x^5$

$= 3584x^5 + 5600x^5$

$= 9184x^5$ **[1mark]**

Hence, the required coefficient is 9184

5	If $(k+3x)^n = 16384 + 86016x + 193536x^2 + \cdots$ Where k and n are real numbers Find the values of k and n **<u>Solution</u>** $(k+3x)^n = k^n + \frac{nk^{n-1}(3x)^1}{1!} + \frac{n(n-1)k^{n-2}(3x)^2}{2!}$ **[1mark]** By comparing the coefficients of the terms in x $(k+3x)^n = 16384 + 86016x + 193536x^2 + \cdots$ $k^n = 16384$---------------------------------A $3nk^{n-1} = 86016$--------------------------B $\frac{9}{2}n(n-1)k^{n-2} = 193536$-------------C **[1mark]** From B $3n\frac{k^n}{k} = 86016$ Substitute $k^n = 16384$ $\frac{3n}{k}16384 = 86016$ $\frac{n}{k} = 1.75$ **[1mark]** Substitute $k^n = 16384$ into equation C $\frac{9}{2}n(n-1)\frac{16384}{k^2} = 193536$ **[1mark]** $n(n-1)\frac{1}{k^2} = 2.625$

	Since $\frac{n}{k} = 1.75$ then $n = 1.75k$ **[1mark]** $\frac{1.75k(1.75k-1)}{k^2} = 2.625$ **[1mark]** $3.0625k - 1.75 = 2.625k$ $0.4375k = 1.75$ $k = 4$ **[1mark]** $n = 1.75k$ $n = 1.75(4)$ $n = 7$ **[1mark]**