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Year 2 (A-Level)

Mixed (sample questions solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1	The graphs of $y = 2x^2 - 5x - 6$ and $y = -\frac{1}{2}x^2 + 3x + 2$ intersect at points A and B a)Calculate the exact coordinates of A and B b) Hence calculate the exact area between the curves **Solution** a)Find the coordinates of A and B $2x^2 - 5x - 6 = -\frac{1}{2}x^2 + 3x + 2$ $5x^2 - 16x - 16 = 0$ $(5x + 4)(x - 4) = 0$ Therefore, $x = -\frac{4}{5}$ or $x = 4$ **[2marks]** When $x = -\frac{4}{5}$ $y = 2\left(-\frac{4}{5}\right)^2 - 5\left(-\frac{4}{5}\right) - 6 = -\frac{18}{25}$ When $x = 4$ $y = 2(4)^2 - 5(4) - 6 = 6$ **[2marks]** Hence the coordinate of A and B are $\left(-\frac{4}{5}, -\frac{18}{25}\right)$ and $(4, 6)$ respectively

b) The area between two curves with intersections at a and b: $\{\int_a^b [f(x) - g(x)]dx\}$

Therefore

$$\int_{-\frac{4}{5}}^{4} \left(-\frac{1}{2}x^2 + 3x + 2\right) dx - \int_{-\frac{4}{5}}^{4} (2x^2 - 5x - 6)dx$$

[1mark]

$$\left[-\frac{x^3}{6} + \frac{3}{2}x^2 + 2x\right]_{-\frac{4}{5}}^{4} - \left[\frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x\right]_{-\frac{4}{5}}^{4}$$ **[2marks]**

$$\frac{2736}{125} - \left(-\frac{3024}{125}\right) = \frac{5760}{125} = \frac{1152}{25}$$ **[1mark]**

Hence, the area between the curves is $\frac{1152}{25}$ square units

2	The third and sixth terms of an A.P. are 15 and 21 respectively a)Find the Nth term of the sequence b)Hence find the sum of the first 12 terms **<u>Solution</u>** a) $a_3 = 15$ $a_6 = 21$ Therefore, $15 = a_1 + 2d$-----Equation 1 $21 = a_1 + 5d$-----Equation 2 **[1mark]** Subtract Equation 1 from Equation 2 $3d = 6$ $d = 2$ **[1mark]** $a_1 + 2d = 15$ $a_1 + 2(2) = 15$ $a_1 = 11$ **[1mark]** So, $a_n = a_1 + (n-1)d$ Hence, $a_n = a_1 + 2(n-1)$ **[1mark]**

b)

$a_1 = 11$

$a_n = a_1 + 2(n-1)$

$a_{12} = 11 + 2(12-1)$

$a_{12} = 33$ **[1mark]**

$S_{12} = \frac{12}{2}(11 + 33)$ **[1mark]**

Therefore, $S_{12} = 264$ **[1mark]**

3

Expand $\left(\frac{1-2x}{1+x}\right)^{\frac{3}{2}}$ up to and including the term in x^3

Solution

$\left(\frac{1-2x}{1+x}\right)^{\frac{3}{2}} = (1-2x)^{\frac{3}{2}}(1+x)^{-\frac{3}{2}}$ **[1mark]**

Expand $(1-2x)^{\frac{3}{2}}$ to the fourth term

$(1-2x)^{\frac{3}{2}} = 1 + \left(\frac{3}{2}\right)(-2x) + \frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2}(-2x)^2 + \frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{6}(-2x)^3$

$(1-2x)^{\frac{3}{2}} = 1 - 3x + \frac{3}{2}x^2 + \frac{1}{2}x^3$ **[2marks]**

Expand $(1+x)^{-\frac{3}{2}}$ to the fourth term

$(1+x)^{-\frac{3}{2}} = 1 + \left(-\frac{3}{2}\right)(x) + \frac{\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{2}(x)^2 + \frac{\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{6}(x)^3$

$(1-x)^{-\frac{3}{2}} = 1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{35}{16}x^3$ **[2marks]**

$\left(\frac{1-2x}{1+x}\right)^{\frac{3}{2}} = \left[1 - 3x + \frac{3}{2}x^2 + \frac{1}{2}x^3\right]\left[1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{35}{16}x^3\right]$

$= 1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{35}{16}x^3 - 3x + \frac{9}{2}x^2 - \frac{45}{8}x^3 + \frac{3}{2}x^2 - \frac{9}{4}x^3 + \frac{1}{2}x^3$

[2marks]

Hence, $\boxed{\left(\frac{1-2x}{1+x}\right)^{\frac{3}{2}} = 1 - \frac{9}{2}x + \frac{63}{8}x^2 - \frac{153}{16}x^3}$ **[1mark]**

4	Given that $\sin A = \frac{2}{3}$ and $\sin B = \frac{5}{6}$

where A and B are acute angles

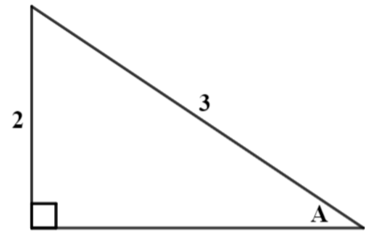
a)Find the exact values of $\cos(A + B)$ and $\sin(A + B)$

b)Hence or otherwise find $\tan(A + B)$

Solution

a)If $\sin A = \frac{2}{3}$

Then we have a right-angled triangle as shown on the right:



Apply Pythagoras theorem,

$3^2 = 2^2 + y^2$ where y is the missing side

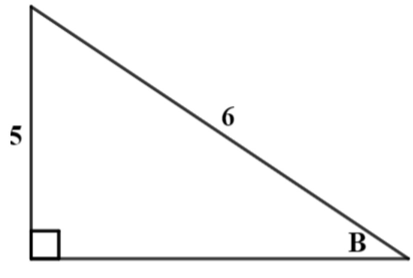
$y^2 = 5$

$y = \sqrt{5}$ **[1mark]**

Therefore, $\cos A = \frac{\sqrt{5}}{3}$ **[1mark]**

If $\sin B = \frac{5}{6}$

Then we have a right-angled triangle as shown on the right:



$y^2 = 6^2 - 5^2$

$y = \sqrt{11}$ **[1mark]**

Therefore, $\cos B = \frac{\sqrt{11}}{6}$ **[1mark]**

	Now use the compound Angle formulas $\cos(\mathrm{A} \pm \mathrm{B}) = \cos\mathrm{A}\cos\mathrm{B} \mp \sin\mathrm{A}\sin\mathrm{B}$ $\sin(\mathrm{A} \pm \mathrm{B}) = \sin\mathrm{A}\cos\mathrm{B} \pm \cos\mathrm{A}\sin\mathrm{B}$ Therefore, $\cos(A+B) = \frac{\sqrt{5}}{3} \times \frac{\sqrt{11}}{6} - \frac{2}{3} \times \frac{5}{6}$ $\cos(A+B) = \frac{\sqrt{55}}{18} - \frac{10}{18}$ Hence, $\cos(A+B) = \frac{-10+\sqrt{55}}{18}$ **[1mark]** Similarly, $\sin(A+B) = \sin A\cos B + \cos A\sin B$ Therefore, $\sin(A+B) = \frac{2}{3} \times \frac{\sqrt{11}}{6} + \frac{\sqrt{5}}{3} \times \frac{5}{6}$ $\sin(A+B) = \frac{\sqrt{11}}{9} + \frac{5\sqrt{5}}{18}$ Hence, $\sin(A+B) = \frac{2\sqrt{11}+5\sqrt{5}}{18}$ **[1mark]** b) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$ Therefore we have $\tan(A+B) = \frac{2\sqrt{11}+5\sqrt{5}}{18} \div \frac{-10+\sqrt{55}}{18}$ **[1mark]** $= \frac{2\sqrt{11}+5\sqrt{5}}{18} \times \frac{18}{-10+\sqrt{55}}$ $= \frac{2\sqrt{11}+5\sqrt{5}}{-10+\sqrt{55}}$ **[1mark]** Hence, $\tan(\mathrm{A}+\mathrm{B}) = -\frac{5\sqrt{11}+8\sqrt{5}}{5}$ **[2marks]**

5	The gradient function of a curve is given as $$f'(x) = \frac{2}{3x} + 4x$$ Given that the curve passes through the point $(1, 0)$ a)Find $f(x)$ b)Hence evaluate $\int_1^4 f(x)\,dx$ Give your answer as an exact value **<u>Solution</u>** a)Integrate $f'(x)$ to find $f(x)$ $$\int f'(x)\,dx = \int \frac{2}{3x}\,dx + \int 4x\,dx$$ **[1mark]** $$f(x) = \frac{2}{3}\ln x + 2x^2 + c$$ **[1mark]** Using (1,0) We have $0 = \frac{2}{3}\ln 1 + 2 \times 1^2 + c$ Hence, $c = -2$ **[1mark]** Therefore, $\boxed{f(x) = \frac{2}{3}\ln x + 2x^2 - 2}$

b) Now integrate f(x) and apply the limits

$$\int_1^4 \mathrm{f}(x)dx = \int_1^4 \frac{2}{3}\ln x\, dx + \int_1^4 2x^2 dx + \int_1^4 (-2)dx$$

Apply integration by parts to the first expression

$$u = \ln x \text{ and } v' = 1$$

$$\int_1^4 \mathrm{f}(x)dx = \frac{2}{3}\left[x\ln x - \int 1dx\right]_1^4 + \frac{2}{3}[x^3]_1^4 - 2[x]_1^4$$

$\int_1^4 \mathrm{f}(x)dx = \frac{2}{3}[x\ln x - x]_1^4 + \frac{2}{3}(64-1) - 2(3)$ **[3marks]**

$\int_1^4 \mathrm{f}(x)dx = \frac{16}{3}\ln 2 - 2 + 42 - 6$

$\int_1^4 \mathrm{f}(x) = \frac{16}{3}\ln 2 + 34$ **[2marks]**