FluidMaths

Year 2 (A-Level)

Binomial Expansion Set 2 – (Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1	Given that the expansion of $(1-2x)^{-\frac{1}{2}}$ is to be used to approximate the value of $\sqrt{\frac{100}{99}}$ What is the value of *x* that would be appropriate? **Solution** $\sqrt{\frac{100}{99}} = \left(\frac{99}{100}\right)^{-\frac{1}{2}}$ **[1mark]** Therefore, $1-2x = \frac{99}{100}$ **[1mark]** $2x = 1 - \frac{99}{100}$ $2x = \frac{1}{100}$ Hence, $x = \frac{1}{200} = 0.005$ **[1mark]**

2	Express $\sqrt{21}$ in the form $k(1-x)^{\frac{1}{2}}$. where k is a constant Hence find an approximate value for $-8\sqrt{21}$ Ignore powers of x beyond x^3 **Solution** $\sqrt{21} = \sqrt{25-4} = \sqrt{25\left(1-\frac{4}{25}\right)}$ $\sqrt{21} = 5(1-\frac{4}{25})^{\frac{1}{2}}$ **[1mark]** $k = 5$ and $x = \frac{4}{25}$ Expand $(1-x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-x)$ $+ \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)(-x)^2}{2}$ $+ \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)(-x)^3}{6}$ $(1-x)^{\frac{1}{2}} = 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16}$ **[3marks]**

Substitute $x = \frac{4}{25}$ into the expansion

$$\sqrt{21} = 5\left(1 - \frac{\frac{4}{25}}{2} - \frac{\left(\frac{4}{25}\right)^2}{8} - \frac{\left(\frac{4}{25}\right)^3}{16}\right)$$ **[1mark]**

$\sqrt{21} = 5(0.917)$

$\sqrt{21} = 4.58$(3sf) **[1mark]**

Hence, $-8\sqrt{21} = -8(4.58)$

$-8\sqrt{21} = -36.7$(3sf) **[1mark]**

3 Expand the expression $(1+3x)^{\frac{1}{2}}(1-2x)^{\frac{1}{3}}$ as far as the term in x^3

Solution

Expand the first expression

$$(1+3x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(3x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(3x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(3x)^3$$

$$= 1 + \frac{3}{2}x - \frac{9x^2}{8} + \frac{27}{16}x^3 \ldots$$

[1mark]

Expand the second expression

$$(1-2x)^{\frac{1}{3}} = \left(1+(-2x)\right)^{\frac{1}{3}}$$

$$\left(1+(-2x)\right)^{\frac{1}{3}} = 1 + \left(\frac{1}{3}\right)(-2x) + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(-2x)^2}{2} + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(-2x)^3}{6}$$

$$= 1 - \frac{2}{3}x - \frac{4x^2}{9} - \frac{40}{81}x^3 \ldots$$

[1mark]

The Expression modifies to:

$$\left(1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3\right)\left(1+\frac{2}{3}x-\frac{4}{9}x^2+\frac{40}{81}x^3\right)$$

Only expand up to the x^3 term

$$=1-\frac{2}{3}x-\frac{4}{9}x^2-\frac{40}{81}x^3+\frac{3}{2}x-x^2-\frac{2}{3}x^3-\frac{9}{8}x^2+\frac{3}{4}x^3+\frac{27}{16}x^3$$

[2marks]

Now simplify

$$\boxed{1+\frac{5}{6}x-\frac{185}{72}x^2+\frac{1655}{1296}x^3\ldots}$$

[1mark]

4	Expand the expression $(1-3x)^n$ where n is a rational number, as far as the term x^3 Use your expansion to find an approximation for $\sqrt{97}$ **Solution** $(1-3x)^n = 1 + n(-3x)$ $+\dfrac{n(n-1)}{2}(-3x)^2$ $+\frac{n(n-1)(n-2)}{6}(-3x)^3$ **[1mark]** $(1-3x)^n = 1 - 3nx + \frac{9n(n-1)}{2}x^2 - \frac{27n(n-1)(n-2)}{6}x^3$ **[1mark]** Therefore, $\sqrt{97} = \sqrt{100-3} = \sqrt{100\left(1-\frac{3}{100}\right)}$**[1mark]** $\sqrt{97} = 10\sqrt{\left(1-\frac{3}{100}\right)}$ $\sqrt{97} = 10\left(1-\frac{3}{100}\right)^{\frac{1}{2}}$ **[1mark]** Therefore, $x = \frac{3}{100}$ and $n = \frac{1}{2}$

Substitute $x = \frac{3}{100}$ and $n = \frac{1}{2}$

$$\left(1 - \frac{3}{100}\right)^{\frac{1}{2}} = 1 - \left(\frac{1}{2}\right)\left(\frac{3}{100}\right)$$
$$+ \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(\frac{3}{100}\right)^2$$
$$+ \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}\left(-\frac{3}{100}\right)^3 \qquad \textbf{[1mark]}$$

$$\left(1 - \frac{3}{100}\right)^{\frac{1}{2}} = 1 - \frac{3}{200} - \frac{9}{8 \times 100^2} - \frac{27}{16 \times 100^3}$$

$$\left(1 - \frac{3}{100}\right)^{\frac{1}{2}} = 0.9898 \qquad \textbf{[1mark]}$$

Therefore, $\sqrt{97} = 10(0.9898)$

$\sqrt{97} = 9.898$ **[1mark]**

5	Given that $-\frac{3}{4}x$ and $\frac{3}{8}x^2$ are part of the expansion of the expression $(1+kx)^n$ Find the values of the rational numbers *k* and *n* **Solution** $(1+kx)^n = 1 + n(kx) + \frac{n(n-1)}{2}(kx)^2$ {Simplify} $= 1 + n(kx) + \frac{n(n-1)k^2}{2}x^2$ **[1mark]** Using the information from the question $kn = -\frac{3}{4}$ and $\frac{n(n-1)k^2}{2} = \frac{3}{8}$ **[1mark]** $4kn = -3$ $k = -\frac{3}{4n}$ $k^2 = \left(-\frac{3}{4n}\right)^2$ Therefore, $k^2 = \frac{9}{16n^2}$ Substitute $k^2 = \frac{9}{16n^2}$ into $\frac{n(n-1)k^2}{2} = \frac{3}{8}$ $\frac{n(n-1)}{2} \times \frac{9}{16n^2} = \frac{3}{8}$ $\frac{9n^2-9n}{32n^2} = \frac{3}{8}$ $72n^2 - 72n = 96n^2$ $-24n^2 - 72n = 0$ $n + 3 = 0$ **[1mark]** Therefore, $n = -3$ **[1mark]** Substitute $n = -3$ into $k = -\frac{3}{4n}$ $k = -\frac{3}{4(-3)} = \frac{1}{4}$ **[1mark]**

6	
	a) Express $\frac{-3x}{(x+2)(1+x)^2}$ in partial fractions b) Hence expand the fractions in ascending powers of x up to and including the term x^3 c) State the validity of your expansion **Solution** a) $\frac{-3x}{(x+2)(1+x)^2} = \frac{A}{(x+2)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$ **[1mark]** $-3x = \mathrm{A}(1+x)^2 + \mathrm{B}(x+2)(1+x) + \mathrm{C}(x+2)$ Let $x = -1$ Then we have $-3(-1) = \mathrm{C}(-1+2)$ Therefore, $\mathrm{C} = 3$ **[1mark]** Let $x = -2$ Then we have $-3(-2) = A(1-2)^2$ Therefore, $\mathrm{A} = 6$ **[1mark]** Let $x = 1$ Then we have $3(1) = 6(1+2)^2 + B(1+2)(1+1) + 3(1+2)$ Therefore, $\mathrm{B} = -6$ **[1mark]** Hence, $\frac{-3x}{(x+2)(1+x)^2} = \frac{6}{(x+2)} - \frac{6}{(1+x)} + \frac{3}{(1+x)^2}$

b) Expand each fraction separately

$$\frac{6}{2+x} = 6(2+x)^{-1} = 6 \times \frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$$

$$6(2+x)^{-1} = 3\left(1-\frac{x}{2}+\frac{-1(-2)\left(\frac{x}{2}\right)^2}{2}+\frac{-1(-2)(-3)\left(\frac{x}{2}\right)^3}{6}\right)$$

[1mark]

$$= 3\left(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}\right)$$

Therefore, $6(2+x)^{-1} = 3 - \frac{3}{2}x + \frac{3}{4}x^2 - \frac{3}{8}x^3$

[1mark]

The expansion is valid if $\left|\frac{x}{2}\right| < 1$

Hence, $|x| < 2$

$$-6(1+x)^{-1} = -6\left(1-x+\frac{-1(-2)x^2}{2}+\frac{-1(-2)(-3)x^3}{6}\right)$$

$$= -6(1-x+x^2-x^3)$$ **[1mark]**

Therefore, $-6(1+x)^{-1} = -6+6x-6x^2-6x^3$

[1mark]

The expansion is valid if $|x| < 1$

$$3(1+x)^{-2} = 3\left(1-2x+\frac{-2(-3)x^2}{2}+\frac{-2(-3)(-4)x^3}{6}\right)$$

$= 3(1-2x+3x^2-4x^3)$ **[1mark]**

$3(1+x)^{-2} = 3-6x+9x^2-12x^3$ **[1mark]**

And this expansion is valid if $|x| < 1$

Hence, $\frac{-3x}{(x+2)(1+x)^2} = \left(3 - \frac{3}{2}x + \frac{3}{4}x^2 - \frac{3}{8}x^3\right)$

$$+(-6 + 6x - 6x^2 - 6x^3)$$

$$+(3 - 6x + 9x^2 - 12x^3)$$

{Simplify the like terms}

$= -\frac{3}{2}x + \frac{15}{4}x^2 - \frac{147}{8}x^3$ **[2marks]**

c) The expansion is valid for $-1 < x < 1$ **[2marks]**