FluidMaths

Year 2 (A-Level)

Algebraic Methods – Set 2 (Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1

Express $\frac{\frac{2}{x}+3x+7}{2x-1}$ as partial fractions

Solution

$$\frac{\frac{2}{x}+3x+7}{2x-1} = \frac{3x^2+7x+2}{x(2x-1)}$$

$$= \frac{3x^2+7x+2}{2x^2+x}$$ **[1mark]**

You may use long division

$$\begin{array}{r} 1.5 \\ 2x^2 - x \,\overline{)\,3x^2 + 7x + 2} \\ \underline{3x^2 - 1.5x} \\ 8.5x + 2 \end{array}$$

[1mark]

Therefore, $\frac{3x^2+7x+2}{2x^2+x} = \frac{3}{2} + \frac{\frac{17x}{2}+2}{2x^2-x}$ **[1mark]**

$$\frac{3}{2} + \frac{\frac{17x}{2}+2}{2x^2-x} = \frac{3}{2} + \frac{17x+4}{2(2x^2-x)}$$ **[1mark]**

Express $\frac{17x+4}{2(2x^2-x)}$ as partial fractions

$$\frac{17x+4}{2(2x^2-x)} = \frac{17x+4}{2x(2x-1)}$$

$$\frac{17x+4}{2x(2x-1)} = \frac{A}{2x} + \frac{B}{2x-1}$$ **[1mark]**

$$17x + 4 = \mathrm{A}(2x-1) + \mathrm{B}(2x)$$

Let $x = 0$

Then we have $4 = -\mathrm{A}$.

Therefore, $\mathrm{A} = -4$ **[1mark]**

Let $x = \frac{1}{2}$

Then we have $17 \times \frac{1}{2} + 4 = \mathrm{B}$

Therefore, $\mathrm{B} = \frac{25}{2}$ **[1mark]**

Hence, $\frac{\frac{2}{x}+3x+7}{2x-1} = \frac{3}{2} - \frac{4}{2x} + \frac{25}{2(2x-1)}$ {Simplify}

$\frac{\frac{2}{x}+3x+7}{2x-1} = \frac{3}{2} - \frac{2}{x} + \frac{25}{2(2x-1)}$ **[1mark]**

2

Show that $\frac{\sqrt{x^3}+x^2\sqrt{x}}{\sqrt{x^3}}$ can be written as $x^n + 1$

Where n is an integer

Solution

Convert the roots to powers

$$\frac{\sqrt{x^3}+x^2\sqrt{x}}{x\sqrt{x^3}} = \frac{\left(x^3\right)^{\frac{1}{2}}+x^{\frac{5}{2}}}{x^{\frac{5}{2}}} =$$ **[1mark]**

$$\frac{x^{\frac{3}{2}}+x^{\frac{5}{2}}}{x^{\frac{3}{2}}} = \frac{x^{\frac{3}{2}}}{x^{\frac{5}{2}}} + \frac{x^{\frac{5}{2}}}{x^{\frac{5}{2}}}$$ **[2marks]**

$$\frac{x^{\frac{3}{2}}}{x^{\frac{5}{2}}} + \frac{x^{\frac{5}{2}}}{x^{\frac{3}{2}}} = x^{-1} + 1$$ **[2marks]**

3	Give that $\frac{px+3}{(x+2)(x^2+4x+5)} = -\frac{1}{x+2} + \frac{qx+r}{x^2+4x+5}$ Find the values of the constants p, q and r **Solution** $px + 3 = -1(x^2 + 4x + 5) + (qx + r)(x + 2)$ $= -x^2 - 4x - 5 + qx^2 + 2qx + rx + 2r$ $= (-1 + q)x^2 + (-4 + 2q + r)x + (2r - 5)$ **[2marks]** Comparing coefficients $-1 + q = 0$ Therefore, $q = 1$ **[2marks]** $2r - 5 = 3$ $2r = 8$ Therefore, $r = 4$ **[2marks]** Substitute $q = 1$ and $r = 4$ into $-4 + 2q + r = p$ $p = -4 + 2(1) + 4$ Therefore, $p = 2$ **[2marks]**

4	Express $\frac{x^2+4}{(x-1)^3}$ in partial fractions **Solution** $\frac{x^2+4}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$ **[1mark]** $x^2 + 4 = \mathrm{A}(x-1)^2 + \mathrm{B}(x-1) + \mathrm{C}$ **[1mark]** $x^2 + 4 = \mathrm{A}(x^2 - 2x + 1) + \mathrm{B}(x-1) + \mathrm{C}$ $x^2 + 4 = \mathrm{A}x^2 + (-2\mathrm{A} + \mathrm{B})x + (\mathrm{A} - \mathrm{B} + \mathrm{C})$ **[1mark]** Now comparing coefficients $\mathrm{A} = 1$ **[1mark]** $-2\mathrm{A} + \mathrm{B} = 0$ $\mathrm{B} = 2\mathrm{A}$ Therefore, $\mathrm{B} = 2$ **[1mark]** $\mathrm{A} - \mathrm{B} + \mathrm{C} = 4$ $\mathrm{C} = 4 - \mathrm{A} + \mathrm{B}$ $\mathrm{C} = 4 - 1 + 2$ $\mathrm{C} = 5$ **[1mark]** Hence, $\frac{x^2+4}{(x-1)^3} = \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{5}{(x-1)^3}$

5 Express $\frac{x^3-x^2-4x+1}{x^2-4}$ in the form $\mathrm{A}(x)+\frac{\mathrm{B}}{x-2}+\frac{\mathrm{C}}{x+2}$

State the values of the constants B and C

Solution

Use long division to find $\mathrm{A}(x)$

$$\begin{array}{r} x-1 \\ x^2-4\,\overline{)\,x^3-x^2-4x+1} \\ \underline{-x^3+0-4x} \\ -x^2+1 \\ \underline{-x^2+4} \\ -3 \end{array}$$

[2mark]

Therefore, we have $\frac{x^3-x^2-4x+1}{x^2-4}=(x-1)-\frac{3}{x^2-4}$

Now express $\frac{-3}{x^2-4}=\frac{\mathrm{B}}{x-2}+\frac{\mathrm{C}}{x+2}$ **[1mark]**

$$-3=\mathrm{B}(x+2)+\mathrm{C}(x-2)$$ **[1mark]**

Let $x=-2$

Then we have $-3=\mathrm{C}(-2-2)$

Therefore, $\mathrm{C}=\frac{3}{4}$ **[1mark]**

Let $x=2$

Then we have $-3=\mathrm{B}(2+2)$

Therefore, $\mathrm{B}=-\frac{3}{4}$ **[1mark]**

Hence, $\frac{x^3-x^2-4x+1}{x^2-4}=x-1-\frac{3}{4(x-2)}+\frac{3}{4(x+2)}$

6	Given that $\frac{p(x)}{x(x+1)} = 1 + \frac{2}{x} - \frac{10}{x+1}$ Find the exact roots of $p(x) = 0$ **Solution** $\frac{p(x)}{x(x+1)} = 1 + \frac{2}{x} - \frac{10}{x+1}$ {Multiply both side by $x(x+1)$} $p(x) = x(x+1) + 2(x+1) - 10x$ **[1mark]** $p(x) = x^2 + x + 2x + 2 - 10x$ $p(x) = x^2 - 7x + 2$ **[1mark]** Now Solve $p(x) = 0$ using the quadratic formula $x^2 - 7x + 2 = 0$ $x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(2)}}{2(1)}$ $x = \frac{7 \pm \sqrt{41}}{2}$ So roots are $x = \frac{7-\sqrt{41}}{2}$ or $x = \frac{7+\sqrt{41}}{2}$ **[2marks]**