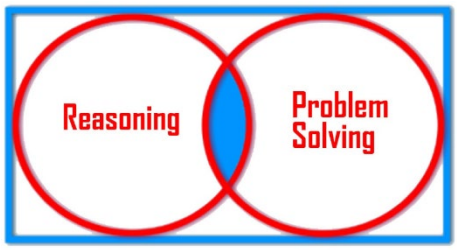

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Year 2 (A-Level)

Vectors-Set 1 (Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 The points $A = (-1, 4, 6); B = (x, 2, 3)$ and $C = (5, y, z)$ are collinear.

a) Find the values of x, y and z

b) Hence calculate the difference in distance between the three points exactly

Solution

a) If the points are collinear, then $k(x, 2, 3) = (-1, 4, 6)$

$2k = 4$

Therefore, $k = 2$

So $2x = -1$

Therefore, $x = -\frac{1}{2}$ **[1mark]**

Also $k\left(-\frac{1}{2}, 2, 3\right) = (5, y, z)$

Therefore, $-\frac{k}{2} = 5$

$k = -10$ **[1mark]**

Hence, $y = 2 \times -10 = -20$

$z = 3 \times -10 = -30$ **[1mark]**

Hence, $A = (-1, 4, 6); B = \left(-\frac{1}{2}, 2, 3\right)$ and $C = (5, -20, -30)$

b) Distance $AB = \sqrt{\left(-\frac{1}{2} - -1\right)^2 + (2-4)^2 + (3-6)^2} = \frac{\sqrt{53}}{2}$ **[1mark]**

Distance $BC = \sqrt{\left(5 - -\frac{1}{2}\right)^2 + (-20-2)^2 + (-30-3)^2} = \frac{11\sqrt{53}}{2}$ **[1mark]**

Difference between AB and BC $= \frac{11\sqrt{53}}{2} - \frac{\sqrt{53}}{2} = 5\sqrt{53}$ **[1mark]**

2 Given that $P = (1, -2, 5); Q = (8, -5, 9)$ and $R = (22, -11, 17)$
Show that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$

Solution

$\overrightarrow{OP} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$; $\overrightarrow{OQ} = \begin{pmatrix} 8 \\ -5 \\ 9 \end{pmatrix}$ and $\overrightarrow{OR} = \begin{pmatrix} 22 \\ -11 \\ 17 \end{pmatrix}$

$\overrightarrow{PQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$

That is, $\overrightarrow{PQ} = -\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \\ 9 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ 4 \end{pmatrix}$ **[1mark]**

$\overrightarrow{QR} = -\overrightarrow{OQ} + \overrightarrow{OR}$

That is, $\overrightarrow{QR} = -\begin{pmatrix} 8 \\ -5 \\ 9 \end{pmatrix} + \begin{pmatrix} 22 \\ -11 \\ 17 \end{pmatrix} = \begin{pmatrix} 14 \\ -6 \\ 8 \end{pmatrix}$ **[1mark]**

If $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ then;

$\overrightarrow{PQ} = k\overrightarrow{QR}$ **[1mark]**

Where k is a constant.

Since $\boxed{\overrightarrow{PQ} = 2\overrightarrow{QR}}$ then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are parallel **[1mark]**

3 The vector $\overrightarrow{OA} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$**.** Given that $\left|\overrightarrow{OA}\right| = 2\sqrt{6}$ and the unit vector in the direction of $\overrightarrow{OA}$ is $\frac{\sqrt{6}}{6}\boldsymbol{i} + \frac{\sqrt{6}}{6}\boldsymbol{j} + \frac{\sqrt{6}}{3}\boldsymbol{k}$**.** Find the values of x, y and z

Solution

The unit vector in the direction of a vector is the vector divided by its magnitude

The unit vector in the direction of $\overrightarrow{OA}$ is $\frac{\overrightarrow{OA}}{|\overrightarrow{OA}|}$

Therefore, $\frac{\sqrt{6}}{6}\boldsymbol{i} + \frac{\sqrt{6}}{6}\boldsymbol{j} + \frac{\sqrt{6}}{3}\boldsymbol{k} = \frac{\overrightarrow{OA}}{2\sqrt{6}}$ **[1mark]**

Therefore, $\overrightarrow{OA} = 2\sqrt{6}\left(\frac{\sqrt{6}}{6}\boldsymbol{i} + \frac{\sqrt{6}}{6}\boldsymbol{j} + \frac{\sqrt{6}}{3}\boldsymbol{k}\right)$

$\overrightarrow{OA} = 2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ **[3marks]**

4 Two vectors are given below

$\boldsymbol{g} = -2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k}$ and $\boldsymbol{h} = 5\boldsymbol{i} + 5\boldsymbol{j} - 12\boldsymbol{k}$

a) Given that $2\boldsymbol{f} + \boldsymbol{g} = 3\boldsymbol{h} - \boldsymbol{g}$ find the column vector **f**

b) Hence calculate the angle between the vectors **f** and **h** to the nearest degree

Solution

a) $2\boldsymbol{f} + \boldsymbol{g} = 3\boldsymbol{h} - \boldsymbol{g}$

Let $f = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ Then

$$2\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix} = 3\begin{bmatrix} 5 \\ 5 \\ -12 \end{bmatrix} - \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}$$ **[1mark]**

$$\begin{bmatrix} 2a \\ 2b \\ 2c \end{bmatrix} + \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -36 \end{bmatrix} - \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 2a \\ 2b \\ 2c \end{bmatrix} = \begin{bmatrix} 19 \\ 7 \\ -50 \end{bmatrix}$$ **[1mark]**

$2a = 19\,, a = \frac{19}{2}$

$2b = 7\,, b = \frac{7}{2}$

$2c = -50\,, c = -25$

$$\boxed{\boldsymbol{f} = \begin{bmatrix} 9.5 \\ 3.5 \\ -25 \end{bmatrix}}$$ **[1mark]**

b) Calculate the magnitude of each vector

$|\boldsymbol{f}| = \sqrt{(9.5)^2 + (3.5)^2 + (-25)^2} = 26.9722$ **[1mark]**

$|\boldsymbol{h}| = \sqrt{(5)^2 + (5)^2 + (-12)^2} = 13.9284$ **[1mark]**

Now we need to find the vector $\overrightarrow{FH}$. This will allow us to use the cosine rule to calculate the angle between the two vectors

$\overrightarrow{FH} = -\overrightarrow{OF} + \overrightarrow{OH}$

That is $\overrightarrow{FH} = -\begin{bmatrix} 9.5 \\ 3.5 \\ -25 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \\ -12 \end{bmatrix} = \begin{bmatrix} -4.5 \\ 1.5 \\ 13 \end{bmatrix}$ **[1mark]**

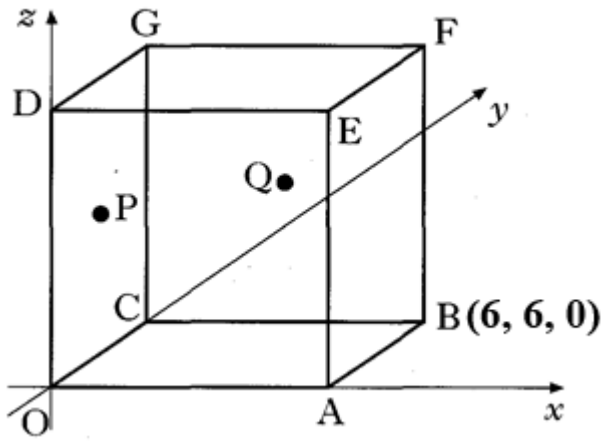
Therefore $\left|\overrightarrow{FH}\right| = \sqrt{(-4.5)^2 + (1.5)^2 + (13)^2} = 13.8384$ **[1mark]**

Let the angle between the vectors **f** and **h** be x. then,

$\cos x = \frac{26.9722^2 + 13.9284^2 - 13.8384^2}{2 \times 26.9722 \times 13.9284} = 0.97157$ **[1mark]**

$x = \cos^{-1}(0.97157) = 14°$ **[1mark]**

5 A cube of side 6 units is shown below



a) Write down the coordinates of F and G

P and Q are the centres on the sides OCGD and GFCB respectively

b) Find the column vectors of $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and hence $\overrightarrow{PQ}$

c) Calculate the angle between $\overrightarrow{OP}$ *and* $\overrightarrow{OQ}$

Solution

a)

$F = (6,6,6)$ **[1mark]**

$G = (0,6,6)$ **[1mark]**

b)

$\overrightarrow{OP} = (0,3,3)$ **[1mark]**

$\overrightarrow{OQ} = (3,6,3)$ **[1mark]**

$\overrightarrow{PQ} = (3,6,3) - (0,3,3)$

$\overrightarrow{PQ} = (3,3,0)$ **[1mark]**

c) Calculate the magnitude of each vector

$|\overrightarrow{OP}| = \sqrt{0^2 + 3^2 + 3^2} = 3\sqrt{2}$ **[1mark]**

$|\overrightarrow{OQ}| = \sqrt{3^2 + 6^2 + 3^2} = 3\sqrt{6}$ **[1mark]**

$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 0^2} = 3\sqrt{2}$ **[1mark]**

Let the angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ be x then

$\cos x = \frac{(3\sqrt{2})^2 + (3\sqrt{6})^2 - (3\sqrt{2})^2}{2 \times 3\sqrt{6} \times 3\sqrt{2}} = \frac{\sqrt{3}}{2}$ **[1mark]**

$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$ **[1mark]**