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Year 2 (A-Level)

Trigonometry – Set 1A
Radians measure, Functions and Graphs
(Solutions)

The marks shown are for guidance purposes only

The questions have been repeated here for your convenience

1 Given that $\theta = 60°$ find the value of θ in radians.

Hence solve the equation $2\cos\theta = 5\sin\theta$ for $0 \le \theta \le 2\pi$

Solution

π Radians $= 180°$

Therefore, $\theta = 60° \times \frac{\pi}{180}$

$\boxed{\theta = \frac{\pi}{3} \text{ rad}}$ **[1Mark]**

The equation can be solved as:

$$2\cos\theta = 5\sin\theta$$

$$\frac{2}{5} = \frac{\sin\theta}{\cos\theta}$$

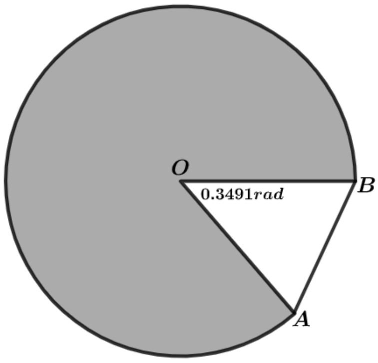
$\frac{2}{5} = \tan\theta$ **[1Mark]**

$$\theta = \tan^{-1}\frac{2}{5}$$

$\boxed{\theta = 0.38 \text{ rad}}$ **[1Mark]**

2 In the diagram below, angle AOB is 0.3491 rads. AB is a straight line

Given that the area of the shaded sector is 249.233 square units



Calculate the perimeter of the shape to 4 decimal places

Solution

Area of sector = $\frac{1}{2}r^2\theta$ where θ is measured in radians

Therefore, $249.233 = \frac{1}{2}r^2\theta$

$\theta = 2\pi - 0.3491 = 5.9341$

So $r = \sqrt{\frac{2\times 249.233}{5.9341}} = 9.1652$ **[1mark]**

{Note that the angle in the shaded sector will be $\theta = 2\pi - 0.3491 = 5.9341$ }

The length of the major arc $AB = 9.1652 \times 5.9341 = 54.3872$ **[1mark]**

Now use the cosine rule to find the length of the side AB

The cosine rule: $\{a^2 = b^2 + c^2 - 2bc \times \cos A\}$

Therefore $(AB)^2 = 9.1652^2 + 9.1652^2 - 2 \times 9.1652 \times 9.1652 \times \cos 0.3491$

$(AB)^2 = 10.1337$ **[1mark]**

Therefore $AB = 3.1833$(4dp) **[1mark]**

Hence the perimeter of the shape will be $= 54.3872 + 3.1833$

$= 57.5705$ units **[1mark]**

3 Given that the angle x rad is small

a) Show that $\frac{1-\cos 3x}{\sin x \tan x} \approx \frac{9}{2}$

b) Find an approximate value for $\sin x \operatorname{cosec} \frac{1}{2}x$

Solutions

a) We need to use small-angle approximations

For small angles in radians

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{1}{2}\theta^2$$

$$\tan\theta \approx \theta$$

Therefore for the numerator

$1 - \cos 3x = 1 - \left(1 - \frac{1}{2} \times (3x)^2\right)$ **[1mark]**

$= \left(1 - \left(1 - \frac{9}{2}x^2\right)\right)$ **[1mark]**

$= \frac{9}{2}x^2$

Hence the numerator: $1 - \cos 3x \approx \frac{9}{2}x^2$ **[1mark]**

For the denominator

$\sin x \approx x$ and $\tan x \approx x$

Therefore, $\sin x \tan x \approx x^2$ **[1mark]**

Hence $\frac{1-\cos 3x}{\sin x \tan x} \approx \frac{9}{2}x^2 \div x^2$ {Apply dividing fractions}

So we have $\frac{9}{2}x^2 \times \frac{1}{x^2} = \frac{9}{2}$ **[1mark]**

Hence $\frac{1-\cos 3x}{\sin x \tan x} \approx \frac{9}{2}$ As Required

b) $\sin x \operatorname{cosec} \frac{1}{2}x \approx x \times \frac{1}{\frac{1}{2}x}$ **[1mark]**

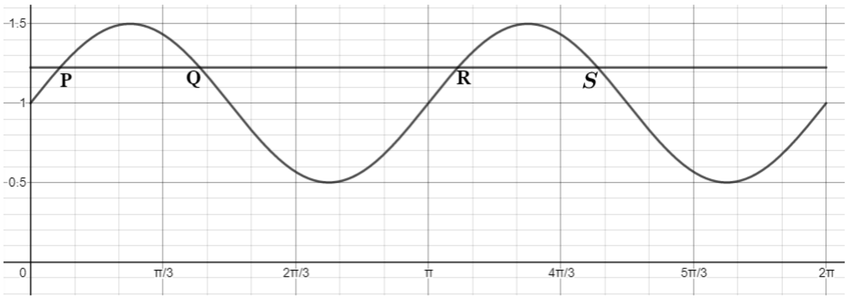
So we have $x \times \frac{2}{x} = 2$

Hence $\sin x \operatorname{cosec} \frac{1}{2}x \approx 2$ **[1mark]**

4	Given that $\tan x = \frac{\sqrt{5}}{6}$ a) Find the exact values of $\sec x$ and $\operatorname{cosec} x$ b) Hence show that $\tan x + \sec x < \operatorname{cosec} x + \sec x$ **<u>Solution</u>** a) $\tan x = \frac{O}{A}$ So we have a right-angled triangle with to angle x as $\sqrt{5}$ and the adjacent as 6 Using Pythagoras theorem: $H^2 = \left(\sqrt{5}\right)^2 + (6)^2 = \sqrt{41}$ **[1mark]** Therefore, $\sec x = \frac{\sqrt{41}}{6}$ **[1mark]** $\csc x = \frac{\sqrt{41}}{\sqrt{5}} = \frac{\sqrt{205}}{5}$ **[1mark]** $\frac{\sqrt{5}}{6} + \frac{\sqrt{41}}{6} = \frac{\sqrt{41}+\sqrt{5}}{6} = 1.4399$ **[1mark]** $\frac{\sqrt{205}}{5} + \frac{\sqrt{41}}{6} = \frac{6\sqrt{205}+5\sqrt{41}}{30} = 3.9308$ **[1mark]** $1.4399 < 3.9308$ Hence, $\tan x + \sec x < \operatorname{cosec} x + \sec x$

5 Part of the graph of $y = \mathrm{f}(x)$ is shown below. where $\mathrm{f}(x) = \mathrm{a}\sin bx + c$

a) Determine the values of a, b and c



On the same grid, part of the graph of $y = \sqrt{\frac{3}{2}}$ is also drawn.

b) The two graphs intersect at the points P, Q, R and S

Determine the coordinates of P, Q, R and S to 4 decimal places

Solution

a) $y = a\sin bx + c$

where |a| = amplitude

$\frac{2\pi}{b}$ = period,

when plotting the graph divide the period into 4 smaller sub-intervals of equal length.

$\frac{-c}{b}$ = phase shift (horizontal shift).

$a = \frac{max - min}{2} = \frac{1.5 - 0.5}{2} = 0.5$

$c = \frac{max + min}{2} = \frac{1.5 + 0.5}{2} = 1$

$b = \frac{2\pi}{period} = \frac{2\pi}{2} = \pi$

The period is the number of full Crust in 360 or 2π

Hence, the equation will be $y = 0.5\sin(\pi x) + 1$ **[3marks]**

	b) $\sqrt{\frac{3}{2}} = 0.5\sin \pi x + 1$ By Solving, we get; $x = \frac{0.46619+2\pi n}{\pi}$ And $x = \frac{n\pi - 0.46619}{\pi}$ Points; $P = (0.1481, 1.2247)$ $Q = (0.8516, 1.2247)$ $R = (2.1484, 1.2247)$ $S = (2.8516, 1.2247$ **[4Marks]**

6

a) Sketch the graph of $y = -\sin x$ on the grid below for $0 \leq x \leq 2\pi$

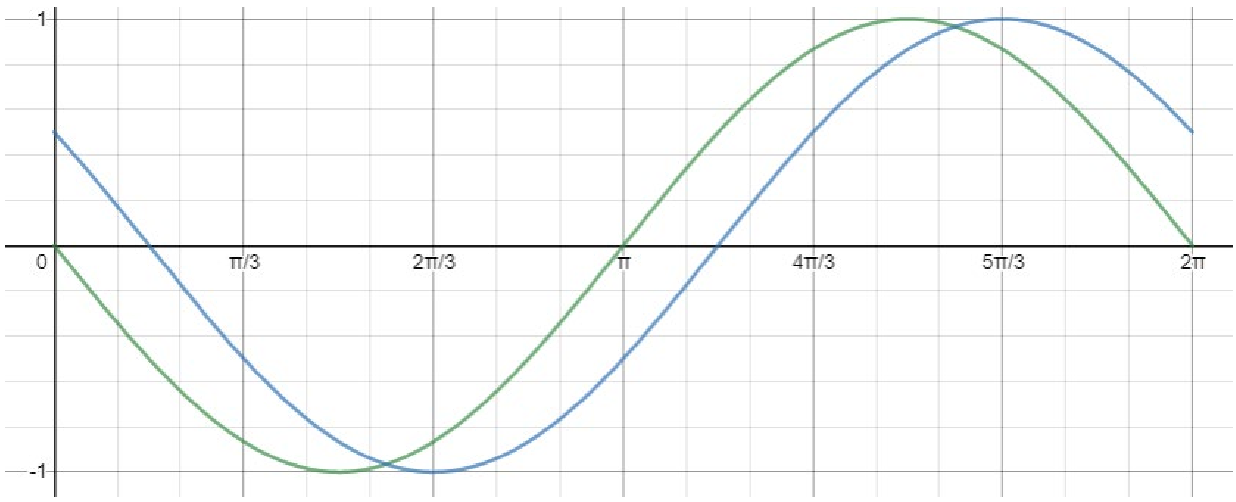
b) On the same grid sketch the graph of $y = \cos\left(x + \frac{\pi}{3}\right)$

c) Use your graph to determine the number of solutions to the equation $-\sin x = \cos\left(x + \frac{\pi}{3}\right)$ in the stated interval and determine the coordinates.

Solution

a) $y = -\sin x$

b) $y = \cos\left(x + \frac{\pi}{3}\right)$ **[2marks]**



c) Two points of intersections so two solutions in the given interval. **[1mark]**

$\text{Cos}\left(x + \frac{\pi}{3}\right) = -\sin x$

$\text{Cos}(x)\,\text{Cos}\left(\frac{\pi}{3}\right) - \sin(x)\sin\left(\frac{\pi}{3}\right) = -\sin x$

$Cos(x) \times 0.5 - \sin(x) \times 0.866 = -\sin x$

$\text{Cos}(x)\,0.5 = -0.134\sin x$

$\tan(x) = -3.713$

from here ;

$x = \pi - 1.3089 = 1.8326$ {$\tan^{-1}(3.713)=1.3089$}

And $x = 2\pi - 1.3089 = 4.9742$

$y = -\sin 1.8326 = -0.966$

$y = -\sin 4.9742 = 0.966$

Hence, the points of intersection are: $(1.8326, -0.966)$ and $(4.9742, 0.966)$ **[4marks]**