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Year 2 (A-Level)

Sequences and Series – Set 1
(Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 The first and last terms of an A.P. are -10 and 55 respectively

a) Given the sum of the terms of the progression is 225, find the number of terms in the sequence.
b) Find the common difference
c) Hence write down the Nth term of the sequence

Solution

a) $a_1 = -10$

$l = 55$

$S_n = 225$

$$S_n = \frac{n}{2}(a_1 + l)$$

$225 = \frac{n}{2}(-10 + 55)$ **[1mark]**

$n = 10$ **[1mark]**

Hence, there are 10 terms in the sequence

b) $a_n = a_1 + (n-1)d$

$a_{10} = a_1 + 9d$

$55 = -10 + 9d$

$d = \frac{65}{9}$ **[2marks]**

c) $a_n = a_1 + (n-1)d$

$a_n = -10 + \frac{65}{9}(n-1)$ **[1mark]**

2	The first 4 terms of an A.P. are given as

$$x, \quad 2x-y, \quad 3x-2y, \quad 4x-3y \ldots$$

a) Find an expression for the 25th term

b) Given that $a_3 = 12$ and $a_5 = 27$ find the values of x and y

c) Find the Nth term of the A.P. and hence find $S_{20} - S_{21}$

Solution

a)

$d = 2x - y - x$

$d = x - y$ **[1mark]**

$a_{25} = a_1 + (25-1)d$

$a_{25} = x + 24(x-y)$

$a_{25} = 25x - 24y$ **[1mark]**

b)

$3x - 2y = 12$

$6x - 4y = 24$------------------(1) **[1mark]**

$27 = x - y + 4x - 3y$

$27 = 5x - 4y$----------(2) **[1mark]**

Solving the two equations gives

$x = -3$ and $y = -\frac{21}{2}$ **[2marks]**

c)

$S_n = \frac{1}{2}n(2a + (n-1)d)$

$d = -3 - -21 = 18$ **[1mark]**

Therefore, $S_{20} = \frac{1}{2} \times 20\big(2(-3) + (20-1)(18)\big) = 3360$ **[1mark]**

$S_{21} = \frac{1}{2} \times 21\,[2(-3) + (19 \times 18)] = 3717$ **[1mark]**

$S_{20} - S_{21} = 3360 - 3717$

Hence, $S_{20} - S_{21} = -357$ **[1mark]**

3 A patient uses 8% of insulin every 15 minutes after its initial administration.

a) An initial dose of 250mg is given. How much insulin is left in the patient after 2 hours?

For the treatment of a certain medical condition, a patient must have at least 150mg but no more than 320mg of insulin in their body at any time t. A doctor prescribes 80mg of insulin per hour for the patient after the initial dose

b) Will the treatment be effective?

Solution

a)

Total amount of insulin = 250mg

Use of insulin in first 15 minutes = 8%

$= 250 \times 8\%$

$= 20\text{mg}$ **[1mark]**

Amount of insulin used in the first 2 hours = $8(20\text{mg})$

$= 160\text{mg}$ **[1mark]**

Amount of insulin left after 2 hours = $250 - 160$

$= 90\text{mg}$ **[1mark]**

b)

Amount of insulin used per hour = 80mg

Insulin injected after 1 hour = 80mg

Insulin left in the body after 1 hour = $250\text{mg} - 80\text{mg} + 80\text{mg}$

$= 250\text{mg}$ **[1mark]**

Therefore, the amount of insulin used is equal to the amount being injected per hour. Hence, the insulin prescription will be effective. **[1mark]**

4	A hotel manager has 120kg of food supplies for his guest. The guest consumes 12% of the food each day but the manager restocks 10kg of food supplies. a) How long will it take before the supplies run below 85kg? b) Will the hotel ever run out of food? Show how you decide **<u>Solution</u>** a) Food stock = 120kg Consumption in one day = 120×12% = 14.4 kg **[1mark]** Addition of stock = 10 kg In one day the reduction in the food stock will be = 4.4kg **[1mark]** $a_1 = 120$ $d = -4.4$ The nth day where stock becomes less than 85kg $a_n = 85$ $a_n = a_1 + (n-1)d$ $85 = 120 + (n-1)(-4.4)$ $n = 8.9$ **[1mark]** Therefore, after 9 days the stocks will be less than 85kg. **[1mark]** b) For the hotel to run out of stock, let $a_n = 0$ $a_1 = 120$ $d = -4.4$ $a_n = a_1 + (n-1)d$ $0 = 120 + (n-1)(-4.4)$ $n = 28.27$ **[1mark]** Hence the stock of food will run out after 29days **[1mark]**

5	The first three terms of a geometric sequence are given as

$$(1+\sqrt{2}), \quad (3+2\sqrt{2}), \quad (7+5\sqrt{2}), ...$$

a) Find the Nth term of the sequence
b) Calculate the exact value of the 5th term
c) Find the value of S_{30}
d) Will the series converge to a limit or diverge? Fully justify your answer

Solution

a)

$$r = \frac{3+2\sqrt{2}}{1+\sqrt{2}}$$

$$r = \frac{3+2\sqrt{2}}{1+\sqrt{2}} \times \frac{1-2\sqrt{2}}{1-\sqrt{2}}$$

$$r = \frac{3-4+2\sqrt{2}-3\sqrt{2}}{1-2}$$

$$r = \frac{-1-\sqrt{2}}{-1}$$

$r = 1+\sqrt{2}$ **[2marks]**

For a geometric sequence: $a_n = a_1 r^{n-1}$

$$a_n = (1+\sqrt{2})(1+\sqrt{2})^{n-1}$$

$a_n = (1+\sqrt{2})^n$ **[1mark]**

b) $a_5 = (1+\sqrt{2})^5$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

$$(1+\sqrt{2})^5 = 1 + \frac{5\sqrt{2}}{1!} + \frac{5\times4(\sqrt{2})^2}{2!} + \frac{5\times4\times3(\sqrt{2})^3}{3!} + \frac{5\times4\times3\times2(\sqrt{2})^4}{4!} + (\sqrt{2})^5$$

$$(1+\sqrt{2})^5 = 1 + 5\sqrt{2} + 20 + 20\sqrt{2} + 20 + 4\sqrt{2}$$

$(1+\sqrt{2})^5 = 41 + 29\sqrt{2}$ **[3marks]**

c) For a geometric series $S_n = \frac{a_1(r^n-1)}{r-1}$

Therefore, $S_{30} = \frac{a_1(r^{30}-1)}{r-1}$

$S_{30} = \frac{(1+\sqrt{2})[(1+\sqrt{2})^{30}-1]}{1+\sqrt{2}-1}$ **[1mark]**

$S_{30} = \frac{(1+\sqrt{2})[(1+\sqrt{2})^{30}-1]}{\sqrt{2}}$

$S_{30} = 5.216 \times 10^{11}$ **[1mark]**

d) The series will diverge since $r > 1$ **[1mark]**

6	The sum to infinity of a G.P. is 25.

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The sum to infinity of a G.P. is 25.
If the second term of the sequence is 2. Find:

a) The Nth term of the sequence.
b) $S_{20} - S_\infty$

Solution

a)

$S_\infty = 25$ $\qquad\qquad$ $a_2 = 2$

For a GP: $S_\infty = \frac{a_1}{1-r}$

$25 = \frac{a_1}{1-r}$---------------(1) **[1mark]**

$a_2 = 2$

$a_1 r = 2$------------------------(2) **[1mark]**

$25 = \dfrac{a_1}{1-r}$

Therefore, $25 = \frac{2/r}{1-r}$

$25 - 25r = \dfrac{2}{r}$

$25r - 25r^2 = 2$

$25r^2 - 25r + 2 = 0$ **[1mark]**

$r = 0.912$ or $r = 0.0878$ **[1mark]**

$a_1 r = 2$

Therefore; $a_1 = 2.193$ or $a_1 = 22.78$ **[1mark]**

Hence there are two possible Nth terms will be as follows

$a_n = 2.193(0.912)^{n-1}$ **[1mark]**

$a_n = 22.78(0.0878)^{n-1}$ **[1mark]**

	b) For series $a_n = 2.193(0.912)^{n-1}$ $S_{20} = \frac{2.193(1-0.912^{20})}{1-0.912}$ $S_{20} = 3.94$ Therefore, $S_{20} - S_{\infty} = 3.94 - 25$ $= -21.06$ **[2marks]** For series $a_n = 22.78(0.0878)^{n-1}$ $S_{20} = \frac{22.78(1-0.0878^{20})}{1-0.0878}$ $S_{20} = 24.97$ Therefore, $S_{20} - S_{\infty} = 24.97 - 25$ $= -0.027$ **[2marks]**