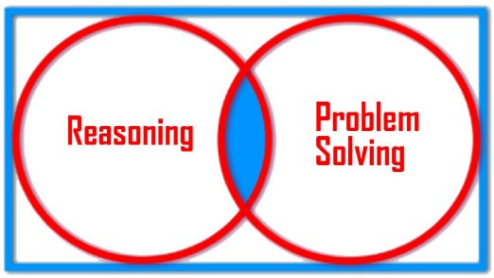

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Year 2 (A-Level)

Integration – Set 1B
(Integration by Substitution and by Parts and Diff Equations)

The marks shown are for guidance purposes only
When not specified, round all non-terminating decimals to 3 significant figures where applicable

Integration

$f(x)$	$\int f'(x)\,dx$
$\int u\frac{dv}{dx}\,dx$	$uv - \int v\frac{du}{dx}\,dx$
$\int \frac{f'(x)}{f(x)}\,dx$	$\ln\lvert f(x)\rvert + c$
$\int \tan x\,dx$	$\ln\lvert\sec x\rvert + c$
$\int \cot x\,dx$	$\ln\lvert\sin x\rvert + c$

1 Given that $u = 1 - e^{3x}$

a) Find $\int \frac{e^{3x}}{\sqrt{1-e^{3x}}}\,dx$ **[5marks]**

b) Hence show that $\int_0^2 \frac{e^{3x}}{\sqrt{1-e^{3x}}}\,dx = -(-1+e^4)^{\frac{1}{2}}$ **[3marks]**

2

a) Show that $2x \sin 2x = 4x \sin x \cos x$ **[2marks]**

b) Hence use integration by parts to prove that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2x \sin 2x \, dx = \frac{7\pi - 3\sqrt{3}}{12}$$

[5marks]

3 Prove that $\int_0^{3\pi} e^{2x} \sin 3x \, dx = \frac{3e^{6\pi}+3}{13}$ **[9marks]**

4 A family of parabolas is defined by the differential equation $dy = (5x - 3)dx$

Two members of the family, f(x) and g(x) pass through the points $(4, 5)$ and $(1, 8)$ respectively.

a) Find the equations for f(x) and g(x) respectively **[6marks]**

b) Hence find the area bound between f(x) and $-g(x)$ **[7marks]**

5 a) Write the express $\frac{3x+4}{x(x+1)}$ as partial fractions **[4marks]**

b) Hence find the general solution for the differential equation $x(x+1)\frac{dy}{dx} = y(3x+4)$ **[5marks]**

c) Hence find the particular solution when $x = 2$ and $y = 3$ **[3marks]**

6 Find the particular solutions for the differential equation

$\text{cosec } x \frac{dy}{dx} = e^x \text{cosec } x + 3x$

when $y = 3$ and $x = \frac{\pi}{3}$ **[8marks]**