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Year 2 (A-Level)

Integration – Set 1B

(Integration by Substitution and by Parts and Diff Equations)

**The marks shown are for guidance purposes only
When not specified, round all non-terminating decimals
to 3 significant figures where applicable**

Integration

$f(x)$	$\int f'(x) dx$
$\int u \frac{dv}{dx} dx$	$uv - \int v \frac{du}{dx} dx$
$\int \frac{f'(x)}{f(x)} dx$	$\ln f(x) + c$
$\int \tan x dx$	$\ln \sec x + c$
$\int \cot x dx$	$\ln \sin x + c$

A series of 25 horizontal lines for writing, arranged in a grid with a vertical margin line on the left side.

2

a) Show that $2x \sin 2x = 4x \sin x \cos x$ [2marks]

b) Hence use integration by parts to prove that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2x \sin 2x dx = \frac{7\pi - 3\sqrt{3}}{12}$$

[5marks]

A blank sheet of lined paper with a vertical margin line on the left side and 30 horizontal ruling lines. The lines are evenly spaced and extend across the width of the page.

3 Prove that $\int_0^{3\pi} e^{2x} \sin 3x dx = \frac{3e^{6\pi}+3}{13}$ **[9marks]**

Lined writing area with 30 horizontal lines.

- 4** A family of parabolas is defined by the differential equation $dy = (5x - 3)dx$
 Two members of the family, $f(x)$ and $g(x)$ pass through the points $(4, 5)$ and $(1, 8)$ respectively.
- a) Find the equations for $f(x)$ and $g(x)$ respectively
[6marks]
 - b) Hence find the area bound between $f(x)$ and $-g(x)$
[7marks]

This image shows a blank sheet of lined paper. It features 30 horizontal lines spaced evenly down the page, intended for writing. The lines are contained within a rectangular border.

5

a) Write the express $\frac{3x+4}{x(x+1)}$ as partial fractions [4marks]

b) Hence find the general solution for the differential equation $x(x+1)\frac{dy}{dx} = y(3x+4)$ [5marks]

c) Hence find the particular solution when $x = 2$ and $y = 3$ [3marks]

Lined writing area consisting of 24 horizontal lines.

6 Find the particular solutions for the differential equation

$$\operatorname{cosec} x \frac{dy}{dx} = e^x \operatorname{cosec} x + 3x$$

when $y = 3$ and $x = \frac{\pi}{3}$ [8marks]

A series of 25 horizontal lines for writing, arranged in a column within a larger rectangular frame.