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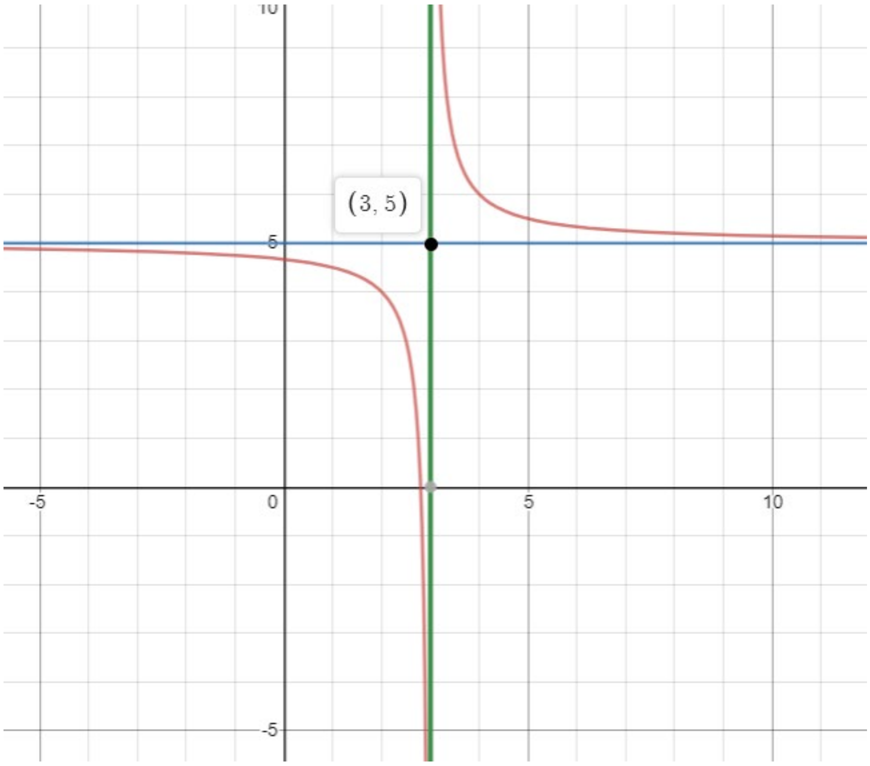
Year 2 (A-Level)

Functions – Set 1 (Solutions)

The marks shown are for guidance purposes only

The questions are repeated here for your convenience

1	Sketch the graph of $y = \frac{1}{x-3} + 5; x \neq 3$ **[2marks]**



Write down the equations of the vertical and horizontal asymptotes and indicate the point where they intersect

Solution

$\frac{1}{x-3} + 5 = \frac{5x-14}{x-3}$ **[1mark]**

The vertical asymptote: $x - 3 = 0$

$x - 3 + 3 = +3$

$x = 3$ **[1mark]**

The horizontal asymptote: Compare coefficients of x

$y = \frac{5x^1-14}{x^1-3} = \frac{5}{1} = \boxed{5}$ **[1mark]**

The intersection at (3, 5)

2	A function f(x) is defined on a set of real numbers as $$f(x) = 4x^2 - 5x + 20$$ a) Show that a possible expression for $f^{-1}(x)$ is $\frac{5+\sqrt{16x-295}}{8}$ b) Another function $g(x)$ is such that $g(x) = 5 - 3x$ Find $\frac{d}{dx} f^{-1}g(x)$ **<u>Solution</u>** a) Interchanging x and y: $x = 4y^2 - 5y + 20$ $4y^2 - 5y + 20 - x = 0$ **[1mark]** $a = 4, \quad b = -5, \quad c = 20 - x$ Using the quadratic formula, $\boxed{f^{-1}(x) = \frac{5+\sqrt{16x-295}}{8} \text{ or } \frac{5-\sqrt{16x-295}}{8}}$ **[2marks]** Hence, $f^{-1}(x) = \frac{5+\sqrt{16x-295}}{8}$ is a possible inverse of f(x) b) $g(x) = 5 - 3x$ $f^{-1}g(x) = \frac{5+\sqrt{16(5-3x)-295}}{8}$ **[1mark]** Hence, $f^{-1}g(x) = \frac{5+\sqrt{-48x-215}}{8}$ **[1mark]** To find $\frac{d}{dx} f^{-1}g(x)$ we need to apply the chain rule.

Factor out the constant term

$\frac{d}{dx}\mathrm{f}^{-1}g(x) = \left(\frac{1}{8}\right)\frac{d}{dx}\left(5 + \sqrt{-48x - 215}\right)$ **[1mark]**

Let $u = -48x - 215$

So $\frac{du}{dx} = -48$ **[1mark]**

So we have $\left(\frac{1}{8}\right)\frac{d}{du}\left(5 + u^{\frac{1}{2}}\right) = \frac{1}{8} \times \frac{1}{2\sqrt{u}}$ **[1mark]**

Therefore, $\frac{d}{dx}\mathrm{f}^{-1}g(x) = \frac{-48}{8(2)\sqrt{-48x-215}}$

$= \frac{-3}{\sqrt{-48x-215}}$ **[1mark]**

3	A function h(x) is given as $h(x) = \frac{2x}{\sqrt{1-x^2}}$ a)State the values of x for which h(x) is undefined b)Find $h^{-1}(x)$ and state its domain **<u>Solution</u>** a)h(x) is undefined for $(-\infty,-1) \cup (1,\infty)$ **[2marks]** b) Interchanging x and y: $x = \frac{2y}{\sqrt{1-y^2}}$ Now solve for y $x\sqrt{1-y^2} = 2y$ **[1mark]** $x^2(1-y^2) = 4y^2$ $x^2 - x^2y^2 = 4y^2$ $x^2 = 4y^2 + x^2y^2$ $y^2(x^2+4) = x^2$ $y^2 = \frac{x^2}{x^2+4}$ **[1mark]** $y = \sqrt{\frac{x^2}{x^2+4}}$ or $-\sqrt{\frac{x^2}{x^2+4}}$ **[1mark]** Neglecting $-\sqrt{\frac{x^2}{x^2+4}}$ So $h^{-1}(x) = \sqrt{\frac{x^2}{x^2+4}} = \frac{x}{\sqrt{x^2+4}}$ **[1mark]** Domain = $(-\infty,\infty)$ **[1mark]**

4 Two functions f and g are defined on a set of real numbers by

f: $x \rightarrow \frac{x-3}{x+4}$; $x \neq -4$ and $g: x \rightarrow \frac{5x}{x-3}$; $x \neq 3$ Find:

a) f$g^{-1}(x)$

b) The image of -2 under f$g^{-1}(x)$

c) Determine whether any of f and g is a one-to-one function

Solution

a)Find $g^{-1}(x)$

$$x = \frac{5y}{y-3}$$

$$x(y-3) = 5y$$

$$xy - 3x = 5y$$

$$xy - 5y = 3x$$

$$y(x-5) = 3x$$

$g^{-1}(x) = \frac{3x}{x-5}$ **[2marks]**

Therefore, $fg^{-1}(x) = f\left(\frac{3x}{x-5}\right)$

$$= \frac{\frac{3x}{x-5}-3}{\frac{3x}{x-5}+4} = \frac{\frac{3x-3(x-5)}{x-5}}{\frac{3x+4(x-5)}{x+5}} = \frac{3x-3x+15}{3x+4x-20}$$ **[2marks]**

Hence, $\boxed{fg^{-1}(x) = \frac{15}{7x-20}}$ **[1mark]**

b) $fg^{-1}(-2) = \frac{15}{7(-2)-20}$

$= -\frac{15}{35}$ **[1mark]**

c)Both functions are one to one since they will pass the horizontal line test

[1mark]

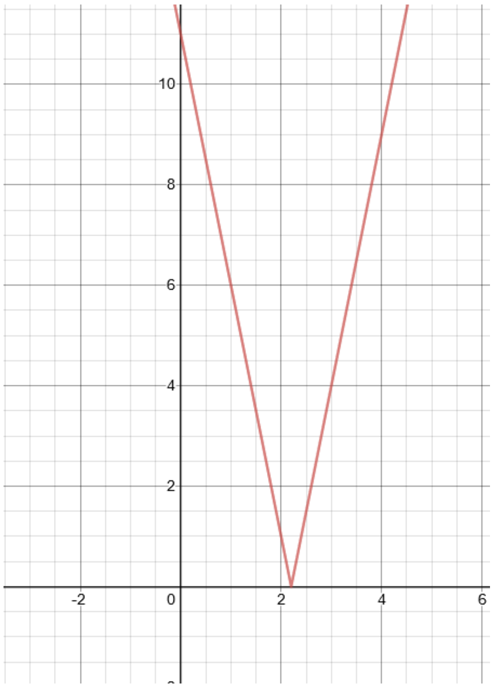
5	Given that $f(x) = 11 - 5x$

a) Sketch the graph of $y = |f(x)|$ and state its domain

b) Hence or otherwise solve the $|f(x)| = |2x|$

Solution

a)



Domain $= -\infty < x < \infty$

$(-\infty, \infty)$ **[3marks]**

b) $|11 - 5x| = |2x|$

$11 - 5x = 2x$

$11 = 7x$

Therefore, $x = \frac{11}{7}$ **[2marks]** or

$11 - 5x = -2x$

$11 = 3x$

Therefore, $x = \frac{11}{3}$ **[2marks]**

6	The equation $\|k-3x\| = \|1+x\|$ has equal roots a)Find the value of k b)Hence solve the equation **<u>Solution</u>** a)Square both sides and use the quadratic discriminant $\|k-3x\| = \|1+x\|$ $(k-3x)^2 = (1+x)^2$ **[1mark]** $k^2 - 6kx + 9x^2 = 1 + 2x + x^2$ $8x^2 + (-6k-2)x + k^2 - 1 = 0$ **[1mark]** $a = 8$, $b = -6k - 2$ and $c = k^2 - 1$ If the roots are equal, then, $b^2 - 4ac = 0$ Therefore, we have $(-6k-2)^2 - 4(8)(k^2-1) = 0$ $(36k^2 + 24k + 4) - (32k^2 + 32) = 0$ $4k^2 + 24k + 36 = 0$ **[1mark]** $k^2 + 6k + 9 = 0$ $k = 3$ **[1mark]** b) Substitute in $k = 3$ into the equation $8x^2 + (-6k-2)x + k^2 - 1 = 0$ $8x^2 + (-6 \times 3 - 2)x + 3^2 - 1 = 0$ $8x^2 - 20x + 8 = 0$ $2x^2 - 5x + 2 = 0$ **[1mark]** Therefore, $x = \frac{1}{2}$ or $x = 2$ **[1mark]**