Year 2

Integration (A – Level)
(Sample Questions Solutions)

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The questions are repeated here for your convenience

1	Given that $$\int_{k}^{3k} 3x\,dx = 48$$ Calculate the possible values of k **<u>Solution</u>** $$\int_{k}^{3k} 3x\,dx = 48$$ $$\left[\frac{3}{2}x^2\right]_{k}^{3k} = 48$$ **[2Marks]** $$\left[\frac{3}{2}(3k)^2\right] - \left[\frac{3}{2}(k)^2\right] = 48$$ $$\frac{27}{2}k^2 - \frac{3}{2}k^2 = 48$$ **[1Mark]** $$12k^2 = 48$$ $$k^2 = 4$$ $$k = \pm 2$$ **[1Mark]**

2	A function is defined on a set of real numbers as $\text{f}(x) = 10x^2 - kx$ Given that $$\int_1^3 \text{f}(x)dx = \frac{116}{3}$$ a) Find the value of k b) Hence evaluate $\text{f}(-6)$ **<u>Solution</u>** a) $$\int_1^3 (10x^2 - kx)dx = \frac{116}{3}$$ $\left[\frac{10x^3}{3} - \frac{kx^2}{2}\right]_1^3 = \frac{116}{3}$ **[2Marks]** $\left[\frac{10(3)^3}{3} - \frac{k(3)^2}{2}\right] - \left[\frac{10}{3} - \frac{k}{2}\right] = \frac{116}{3}$ **[1Mark]** $90 - \frac{9k}{2} - \frac{10}{3} + \frac{k}{2} = \frac{116}{3}$ **[1Mark]** $\frac{-9k+k}{2} = \frac{116}{3} + \frac{10}{3} - 90$ $\frac{-8k}{2} = -48$ $-4k = -48$ $k = 12$ **[1Mark]** Hence, $\text{f}(x) = 10x^2 - 12x$ b) $\text{f}(-6) = 10x^2 - 12x$ $= 10(-6)^2 - 12(-6)$ $= 432$ **[1Mark]**

3 A function is defined on a set of real numbers as $f(x) = x^2 - 4x$

a) Illustrate

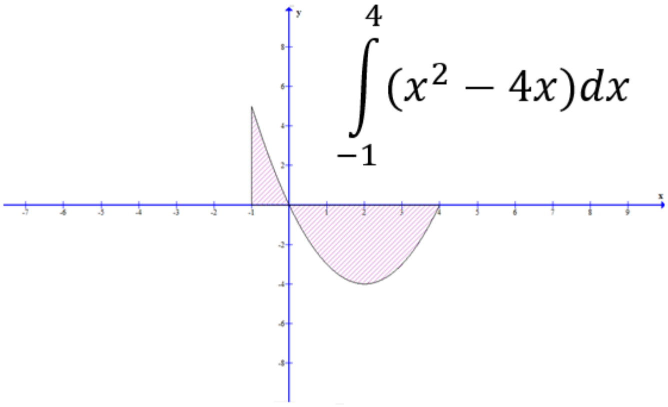
$$\int_{-1}^{4} (x^2 - 4x)dx$$

on a sketch of f(x)

b) Hence evaluate $\int_{-1}^{4} f(x)dx$

Solution

a) This is the area between -1 and 4 on the graph **[3Marks]**



b)

$$\int_{-1}^{4} (x^2 - 4x)dx = \left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_{-1}^{4}$$

[2Marks]

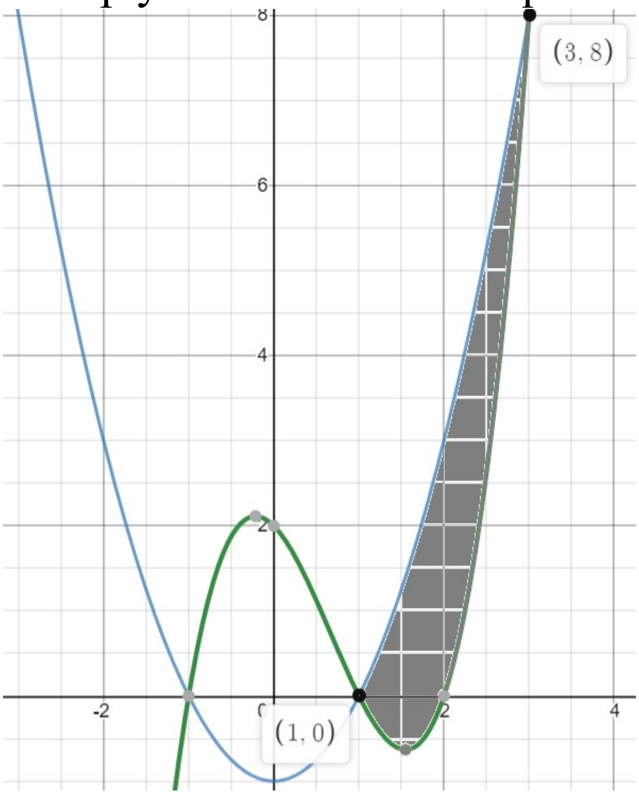
$$\int_{-1}^{4} (x^2 - 4x)dx = \left[\frac{(4)^3}{3} - \frac{4(4)^2}{2}\right] - \left[\frac{(-1)^3}{3} - \frac{4(-1)^2}{2}\right]$$

$= \frac{64}{3} - 32 + \frac{1}{3} + 2$ **[1Mark]**

$\int_{-1}^{4}(x^2 - 4x)dx = -\frac{25}{3}$ **[1Mark]**

4	The functions $y = x^3 - 2x^2 - x + 2$ and $y = x^2 - 1$ intersect at the points A and B. Where A and B are in the first quadrant a) Find the coordinates of A and B b) Hence show that the area bound between the points A and B is a whole number **<u>Solution</u>** a) Set the two equations equal and solve to find x $x^3 - 2x^2 - x + 2 = x^2 - 1$ $x^3 - 3x^2 - x + 3 = 0$ $x^2(x-3) - 1(x-3) = 0$ $(x-3)(x^2-1) = 0$ Therefore, $x = 3, \pm 1$ **[2Marks]** Since both A and B are in the first quadrant, we ignore $x = -1$ When $x = 3$ Then $y = (3)^2 - 1 = 8$ Therefore, $A = (3, 8)$ **[1Mark]** When $x = 1$ Then $y = (1)^2 - 1 = 0$ Therefore, B= $(1, 0)$ **[1Mark]**

b) A sketch will help you visualise the required area



$$\int_1^3 (x^2 - 1)\,dx - \int_1^3 (x^3 - 2x^2 - x + 2)\,dx$$

$$= \left[\frac{x^3}{3} - x\right]_1^3 - \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x\right]_1^3$$

$$= \left[6 - -\frac{2}{3}\right] - \left[\frac{15}{4} - \frac{13}{12}\right]$$

$$= \left[\frac{20}{3} - \frac{8}{3}\right] = 4$$

Hence the area between A and B is 4sq units as required

[4Marks]

5	Given that, $u = 2x^3 + 3$ evaluate the integral below $$\int_{-1}^{\frac{1}{2}} x^2(2x^3+3)^4 dx$$ **<u>Solution</u>** Since $u = 2x^3 + 3$ Then, $\frac{du}{dx} = 6x^2$ **[1Mark]** Therefore, $du = 6x^2\mathrm{d}x$ $x^2\mathrm{d}x = \frac{1}{6}du$ **[1Mark]** At $x = -1 \quad u = -2 + 3 = 1$ **[1Mark]** At $x = \frac{1}{2} \quad u = \frac{2}{8} + 3 = \frac{13}{4}$ **[1Mark]** Therefore, $$\int_{-1}^{\frac{1}{2}} x^2(2x^3+3)^4 dx = \int_{1}^{\frac{13}{4}} \left(\frac{1}{6}\right) u^4\, du$$ $$= \frac{1}{6}\left[\frac{u^5}{5}\right]_1^{\frac{13}{4}}$$ **[1Mark]** $$= \frac{1}{6}\left[\frac{\left(\frac{13}{4}\right)^5}{5}\right] - \frac{1}{6}\left[\frac{1}{5}\right]$$ **[1Mark]** $$= \frac{1}{6}\left(\frac{371293}{1024} \times \frac{1}{5}\right) - \frac{1}{30}$$ $$\frac{371293}{30720} - \frac{1}{30} = 12.1(3\text{sf})$$ **[1Mark]**