Year 1 (AS)

Differentiation
(Sample Questions Solutions)

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The questions are repeated here for your convenience

1	Evaluate $\lim_{x \to 7} \frac{x^2-49}{2x^2-11x-21}$ **<u>Solution</u>** Notice that as x approaches 7, both the numerator and denominator will tend to zero which will make the whole expression undefined Therefore, to evaluate this expression, we need to simplify it first $\frac{x^2-49}{2x^2-11x-21} = \frac{(x-7)(x+7)}{(2x+3)(x-7)}$ {Cancel out the common factors} **[2Marks]** $= \frac{x+7}{2x+3}$ Hence, $\lim_{x \to 7} \frac{x+7}{2x+3}$ tends to $\frac{7+7}{2(7)+3} = \frac{14}{17}$ **[2Marks]**
2	Two functions are defined on a set of real numbers as $g(x) = ax + 7$ and $h(x) = 5x - a$ where a is a constant a) Given that $f'[gh(x)] = 25$, find the value of a b) Hence, evaluate $hg(-2)$ **<u>Solution</u>** a) $g\big(h(x)\big) = a(5x - a) + 7$ $g(h(x)) = 5ax - a^2 + 7$ **[1Mark]** Therefore, $f'[gh(x)] = 5a$ **[1Mark]** Hence, $5a = 25$ Therefore, $a = 5$ **[1Mark]** b) $h(g(x)) = 5(ax + 7) - a$ $= 5ax + 35 - a$ $h(g(-2)) = 5a(-2) + 35 - a$ $= -10a + 35 - a$ $= -11a + 35$ **[1Mark]** Now sub $a = 5$ into the resulting function $h(g(-2)) = -11(5) + 35$ $= -55 + 35$ $= -20$ **[1Mark]**

3	You are given the following information about the equation of a certain curve, $\frac{d^2y}{dx^2} = -12$, $\left.\frac{dy}{dx}\right\|_{x=-2} = 27$ and the coordinate $(-1, 15)$ lies on the graph of the curve. a) Find the equation of the curve b) Hence, find y when $x = \frac{1}{3}$ **Solution** a) Since $\frac{d^2y}{dx^2}$ is a constant, it means that, the equation is a quadratic of the form $ax^2 + bx + c$ Therefore, $\frac{dy}{dx} = 2ax + b$ and $\frac{d^2y}{dx^2} = 2a$ **[2Marks]** Therefore, $2a = -12$ Hence, $a = -6$ **[1Mark]** Since $\left.\frac{dy}{dx}\right\|_{x=-2} = 27$ then, $2a(-2) + b = 27$ That is, $-4a + b = 27$ Now substitute $a = 6$ into $-4a + b = 27$ $-4(-6) + b = 27$ $24 + b = 27$ Therefore, $b = 3$ **[1Mark]** Since $(-1, 15)$ lies on the curve, then $(-6)(-1)^2 + (3)(-1) + c = 15$ $c = 24$ **[1Mark]** Hence the equation of the curve will be $y = -6x^2 + 3x + 24$ **[1Mark]** b) $y = -6\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 24 = \frac{73}{3}$ **[1Mark]**

4	The function $f(x) = 1 - Ax - x^2 + x^3$ has a tangent at the point $(-1,1)$. The tangent passes through the point $(-3,-5)$ a) Find the value of A b) Hence, find the equation of a normal to $f(x)$ at $(2,1)$ **<u>Solution</u>** a) The tangent passes through the points $(-1,1)$ and $(-3,-5)$ Use these to calculate the gradient of the tangent as follows $m_{Tangent} = \frac{-5-1}{-3--1} = \frac{-6}{-2} = 3$ **[1Mark]** This means, the gradient of the curve at the point $(-1,1)$ will be 3 Now differentiate f(x) to find the gradient function $f'(x) = -A - 2x + 3x^2$ **[1Mark]** Therefore, $f'(-1) = 3$ So we have, $-A - 2(-1) + 3(-1)^2 = 3$ **[1Mark]** $-A + 2 + 3 = 3$ Hence, $A = 2$ **[1Mark]** Therefore, $f'(x) = -2 - 2x + 3x^2$ b) Calculate the gradient of the function at $x = 2$ as follows: $f'(x) = -2 - 2x + 3x^2$ $f'(2) = -2 - 2(2) + 3(2)^2$ $f'(-2) = 6$ **[1Mark]** Therefore, the gradient of the normal will be $-\frac{1}{6}$ **[1Mark]** $\{\text{Gradient of Perpendicular lines has a product of} - 1\}$ So, the equation of the normal will be in the form, $y = -\frac{1}{6}x + c$ Now substitute the coordinate (2, 1) into $y = -\frac{1}{6}x + c$ So we have, $1 = -\frac{1}{6}(2) + c$ $3 = -1 + 3c$ $c = \frac{4}{3}$ Hence the equation of the normal will be $y = -\frac{1}{6}x + \frac{4}{3}$ **[1Mark]**

5	A function is given as $f(x) = x^3 + ax^2 + bx + c$ When f(*x*) is divided by $(x - 2)$ and $(x + 1)$ remainders of 8 and 9 are obtained respectively. Given that $f'(-2) = 39$, calculate the values of *a, b* and *c* **<u>Solution</u>** $f(x) = x^3 + ax^2 + bx + c$ $f'(x) = 3x^2 + 2ax + b$ $f'(-2) = 3(-2)^2 + 2a(-2) + b$ $f'(-2) = 12 - 4a + b$ $39 = 12 - 4a + b$ $-4a + b = 27$ − − − − − − − − − − − −Equation (A) **[1Mark]** Substitute $x = 2$ into f(*x*) $f(2) = 2^3 + a(2)^2 + b(2) + c$ $8 = 8 + 4a + 2b + c$ $-4a - 2b = c$ − − − − − − − − − − − −Equation (B) **[1Mark]** Substitute $x = -1$ into f(*x*) $f(-1) = (-1)^3 + a(-1)^2 + b(-1) + c$ $9 = -1 + a - b + c$ $10 = a - b + c$ $c = 10 - a + b$ − − − − − − − − Equation (C) **[1Mark]** Now Equate (B) and (C). That is, $10 - a + b = -4a - 2b$ Therefore, $3a + 3b = -10$ − − − − − − − − Equation (D) Multiply equation (A) by 3 to obtain $-12a + 3b = 81$ Now subtract this from equation D to obtain $3a + 3b = -10$ $\underline{-12a + 3b = 81}$ $15a = -91$ $a = -\frac{91}{15}$ **[1Mark]**

	Substitute $a = -\frac{91}{15}$ into equation (A) $-4\left(-\frac{91}{15}\right) + b = 27$ $b = 27 - \frac{364}{15} = \frac{41}{15}$ Therefore, $b = \frac{41}{15}$ **[1Mark]** Substitute $a = -\frac{91}{15}$ and $b = \frac{41}{15}$ into Equation (C) $c = 10 - \left(-\frac{91}{15}\right) + \left(\frac{41}{15}\right) = \frac{94}{5}$ Therefore, $c = \frac{94}{5}$ **[1Mark]**