GCSE Mathematics (Grade 9-1)

Problem Solving – Sample 3 (Solutions)

H

Reasoning

Problem Solving

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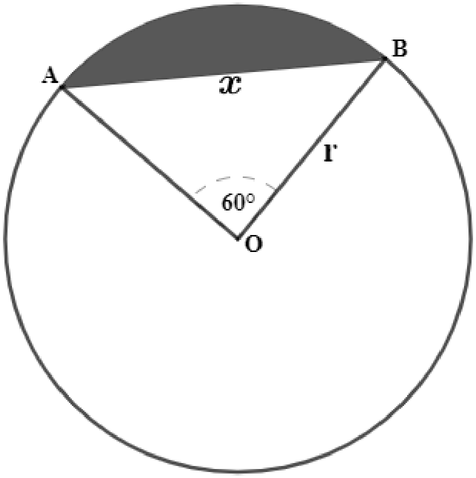
{Aimed at students working towards Grade 9 or 8}

The grades and marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 The diagram below shows a circle with centre O and radius *r*.
The interior angle of the sector is 60°



Show that the area of the shaded segment can be written as $\frac{r^2}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

Solution

We need to find the difference between the area of the sector with radius *r* and the area of triangle AOB.

$\left\{\text{Area of a sector} = \frac{\theta}{360} \times \pi r^2\right\}$ $\left\{\text{Area of a triangle} = \frac{1}{2}ab \sin C\right\}$

Therefore,

Area of the sector $= \frac{60}{360} \times \pi r^2$

$= \frac{\pi r^2}{6}$ **[1mark]** $\left\{\text{Note that } \frac{60}{360} = \frac{1}{6}\right\}$

Area of the triangle $= \frac{1}{2} \times r \times r \times \sin 60$

$= \frac{1}{2} r^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}r^2}{4}$ **[2marks]**

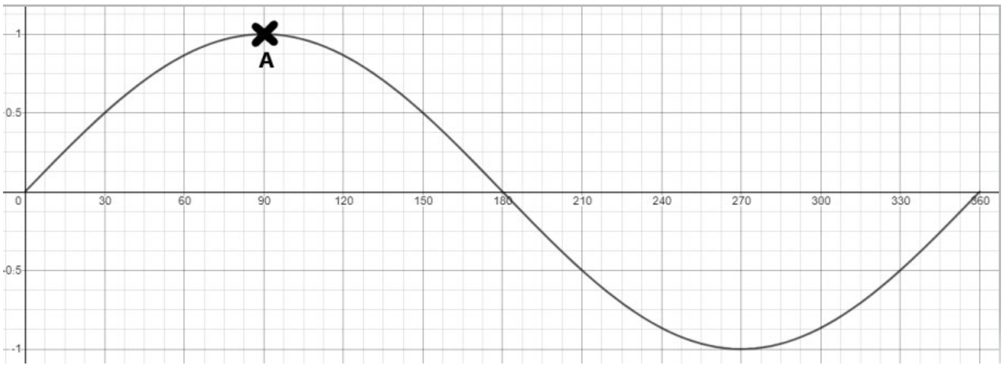
{Note that the exact value of $\sin 60 = \frac{\sqrt{3}}{2}$}

Therefore, the area of the shaded region will be $= \frac{\pi r^2}{6} - \frac{\sqrt{3}r^2}{4}$

{Factorise out $\frac{r^2}{2}$} **[1mark]**

Hence we have, Area of the segment $= \frac{r^2}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ **[1mark]**

2 Part of the graph of the function $f(x) = \sin x$ is shown below
The point A is the maximum point of the graph.



a) Write down the coordinates of the maximum point of the graph of the function $y = f(x - 50)$

b) The maximum point of a transformation of $f(x)$ is given as $(-30, -3)$. Write down the equation of the transformed graph

Solution

a) $y = f(x - 50)$

The transformation shows a 50° shift to the right

Therefore, the coordinates of A will become $(140, 1)$ **[1mark]**

{Note that the *y* coordinate will not change since there has not been any vertical transformation of the graph}

b) Notice that both the *x* and *y* coordinates of the maximum point of the transformation has changed from those of the original.

Therefore, the graph as undergone a combination of transformations.

The *x* coordinate has gone from 90° on the original graph to $-30°$ representing a horizontal shift of 120° to the left.

The *y* coordinate has gone from 1 on the original graph to -3 representing a vertical shift of 4 units in the negative *y* direction.

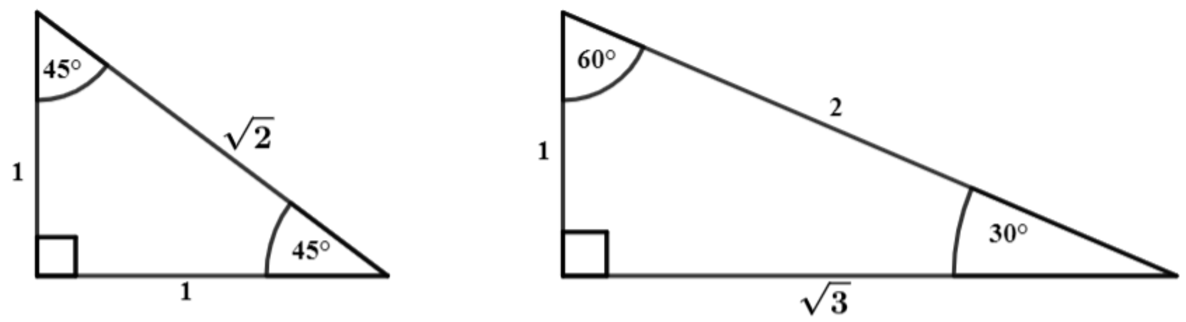
Therefore, the equation of the new graph will be $y = f(x + 120) - 4$

[2marks]

3 Show that $\tan 60 \div \sin 45 = \sqrt{6}$

Solution

We need the exact values of the given angles.
Refer to the triangles in the diagram below:



$\tan 60 = \sqrt{3}$

$\sin 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ {Notice that, after rationalising $\frac{1}{\sqrt{2}}$ we obtain $\frac{\sqrt{2}}{2}$}

Therefore,

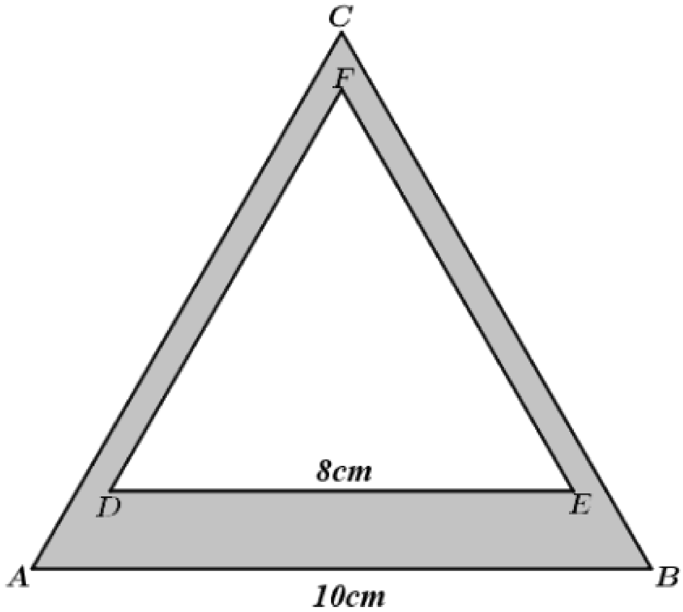
$\tan 60 \div \sin 45 = \sqrt{3} \div \frac{\sqrt{2}}{2}$ **[2marks]**

{Dividing fractions: Keep, Change and Flip}

$= \sqrt{3} \times \frac{2}{\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}}$ {Now rationalise} **[1mark]**

Therefore, we have: $\frac{2\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{6}}{2} = \sqrt{6}$ **[1mark]**

4 Two equilateral triangles, ABC and EDF are shown below
The side length of triangle ABC is 10cm
The side length of triangle EDF is 8cm



Calculate the exact size of the shaded area.

Solution

The shaded area is the difference between the areas of the two triangles {Area of triangle $= \frac{1}{2}ab\sin C$}

Therefore, Area of triangle ABC $= \frac{1}{2} \times 10 \times 10 \times \sin 60$ **[1mark]**

$= \frac{1}{2} \times 100 \times \frac{\sqrt{3}}{2} = \frac{100\sqrt{3}}{4} = 25\sqrt{3}$ **[1mark]**

{Note that, the exact value of $\sin 60 = \frac{\sqrt{3}}{2}$}

Area of triangle EDF $= \frac{1}{2} \times 8 \times 8 \times \sin 60$ **[1mark]**

$= \frac{1}{2} \times 64 \times \frac{\sqrt{3}}{2} = \frac{64\sqrt{3}}{4} = 16\sqrt{3}$ **[1mark]**

Hence, the shaded area $= 25\sqrt{3} - 16\sqrt{3} = 9\sqrt{3}$ **[1mark]**

5	Solve the equation $\sqrt{5} \times 25^x = \frac{1}{125}$ Give your answer as an exact value **<u>Solution</u>** We need to write each term of the equation as a power of 5 Therefore, $5^{\frac{1}{2}} \times 5^{2x} = 5^{-3}$ **[1mark]** {Apply the negative index law: where $\frac{1}{a} = a^{-1}$therefore $\frac{1}{125} = 5^{-3}$} $5^{\frac{1}{2}+2x} = 5^{-3}$ **[1mark]** {Apply the multiplication law of indices: $a^x \times a^y = a^{x+y}$} Therefore, $\frac{1}{2} + 2x = -3$ **[1mark]** $1 + 4x = -6$ $4x = -7$ Therefore, $x = -\frac{7}{4}$ **[1mark]**