GCSE Mathematics (Grade 9-1)

Problem Solving – Sample 2 (Solutions)

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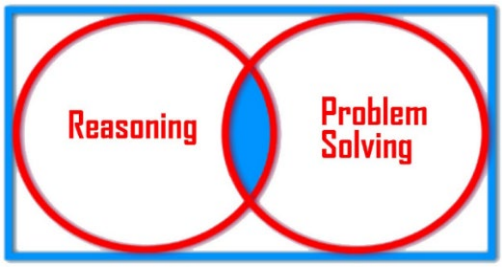

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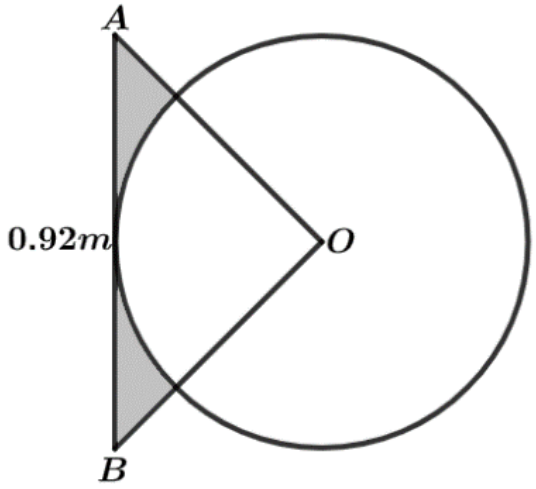
{Aimed at students working towards Grade 9 or 8}

The Grades and Marks shown are for guidance purposes only

The questions are repeated here for your convenience

1 A circle of centre O and area 1.5m^2 is shown below.
In triangle ABO, $\text{AO} = \text{BO} = 0.88\text{m}$ and $\text{AB} = 0.92\text{m}$



Calculate the area of the shaded region

Solution

Use the cosine rule to find angle AOB: $\left\{\cos A = \frac{b^2+c^2-a^2}{2bc}\right\}$

$$\cos \text{AOB} = \frac{(0.88)^2+(0.88)^2-(0.92)^2}{2\times 0.88\times 0.88} = 0.454$$

Therefore, Angle $\text{AOB} = 63°$ to the nearest degree. **[2marks]**

Now we can calculate the area of triangle AOB:
{Area of a triangle $= \frac{1}{2}ab \times \sin C$}
Area of triangle AOB $= \frac{1}{2} \times 0.88 \times 0.88 \times \sin 63 = 0.345\text{m}^2$ **[1mark]**

Now we need to calculate the area of the sector:
{Area of a sector $= \frac{\theta}{360} \times \pi r^2$}
Therefore, $\frac{63}{360} \times 1.5 = 0.263$ (3sf) **[1mark]**
{Note that area of the circle is given as 1.5m^2}

Area of the shaded region $= 0.345 - 0.263 = 0.082\text{m}^2$ **[1mark]**

2 Given that $\frac{2}{3} \times \frac{5}{p} = \frac{1}{2} + \frac{6p}{7}$

Find the possible values of p

Give your answers to 1 decimal place

Solution

Simplify each side of the equation first

$\frac{2}{3} \times \frac{5}{p} = \frac{1}{2} + \frac{6p}{7}$

$\frac{10}{3p} = \frac{7+12p}{14}$ {Cross multiply to remove the fraction} **[2marks]**

$140 = 21p + 36p^2$ {Rearrange}

$36p^2 + 21p - 140 = 0$ **[1mark]**

Solve using the quadratic formula

$a = 36$

$b = 21$

$c = -140$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$p = \frac{-21 \pm \sqrt{21^2 - 4 \times 36 \times -140}}{2 \times 36}$ **[1mark]**

Therefore, $p = 1.7$(1dp) or $p = -2.3$(1dp) **[1mark]**

3 A boat can travel 36miles upstream in 3 hours.
The same boat travelled 40miles downstream in 2 hours.
Calculate the speed of both the boat and current.

Solution

This problem can be solved using simultaneous equations.
Let the speed of the boat be s and the speed of the current be c.
Speed of the boat and current upstream will be $36 \div 3 = 12$mph **[1mark]**

{Speed = Distance ÷ Time}
Speed of the boat and current downstream will be 40 ÷ 2 = 20mph **[1mark]**

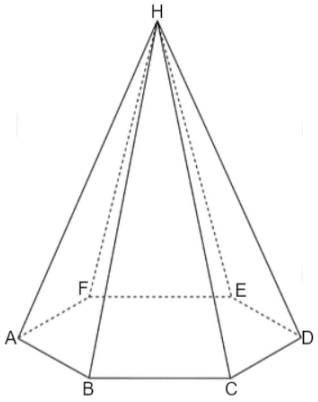
1. Whilst the boat travelled upstream, the currents from the water will be travelling downstream and will cause a drag. Therefore, $s - c = 12$------Equation 1
2. Whilst the boat travelled downstream, the current travelled downstream as well. Therefore, $s + c = 20$------Equation 2

Add Equations 1 and 2 together
$s - c = 12$
$\underline{s + c = 20}$
$2s = 32$
Therefore, $s = 16$ **[1mark]**

Substitute $s = 16$ into Equation 2
So, we have $16 + c = 20$
Therefore, $c = 4$ **[1mark]**

Hence, the speed of the boat and the speed of the current are 16mph and 4mph, respectively.

4 A hexagonal based pyramid is shown below
Where HA = HB = HC = HD = HE = HF = 7cm



The area of the base is 30cm^2 and the volume of the pyramid is 65cm^3
Calculate the angle between the side HD and the base of the pyramid.
Give your answer to the nearest degree

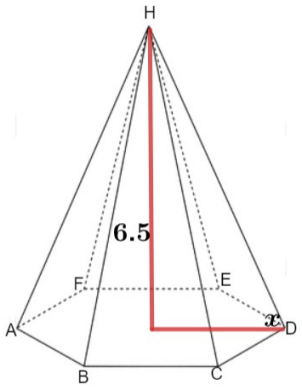
Solution

$\{\text{Volume of a Pyramid} = \frac{1}{3}\ (\text{Area of base})\ \times \text{height}\}$

Therefore, $65 = \frac{1}{3} \times 30 \times h$ {Where h is the height of the pyramid}

So, $65 = 10h$

$h = 6.5\text{cm}$ **[2marks]**



Using SOHCAHTOA, we can find that

$\sin x = \frac{6.5}{7}$ where x is the angle between HD and the base of the pyramid **[1mark]**

$x = \sin^{-1}\left(\frac{6.5}{7}\right) = 68.2°$

Therefore, the angle between the side HD and the base is 68° to the nearest degree. **[1mark]**

5 The events A and B are such that, $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{6}$

a) Given that A and B are mutually exclusive calculate $P(A \text{ or } B)$

b) In fact, A and B are not mutually exclusive. Calculate $P(A \text{ or } B)$

Solution

a) Since the events are mutually exclusive, it means that there is no intersection between them. Therefore $P(A \text{ or } B) = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ **[2marks]**

b) If A and B are not mutually exclusive, then
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **[1mark]**

$= \frac{1}{5} + \frac{1}{6} - \left(\frac{1}{5} \times \frac{1}{6}\right)$ **[1mark]**

$= \frac{11}{30} - \frac{1}{30} = \frac{10}{30}$

Therefore, P(A or B) $= \frac{1}{3}$ **[1mark]**