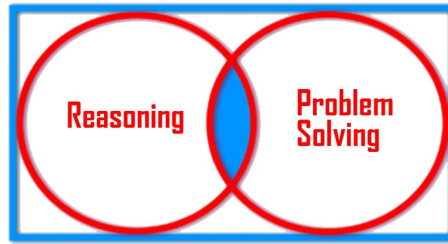


## GCSE Mathematics (Grade 9-1)

# H



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### Problem Solving – Sample Questions

{Aimed at students working towards a Grade 9 or 8}

**The grades and marks shown are for guidance Purposes only**

You may NOT use a Calculator for these set of questions



### Strategies to help you in Problem Solving

- Read the question carefully
- Sketch a diagram where applicable
- Take note of key information
- Write down any formulae you may need
- Tackle the problem in bite size rather than as a whole
- Concentrate on the parts of the problem that make sense to you and try to solve those first
- Collaborate with a partner and share ideas
- Use a dictionary to find the meaning of any confusing words
- Check that your answers make sense in the context of the question

#### Some Useful Formulae

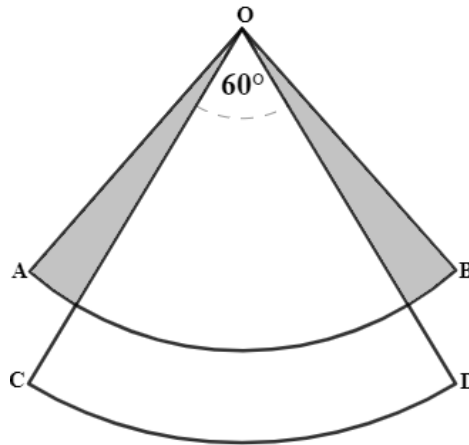
$$\text{Area of a Triangle} = \frac{1}{2} ab \sin C$$

$$\text{The quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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- 2 The sector OCD was formed from a circle with equation  $x^2 + y^2 = 64$ .  
The ratio of the length of OC to the length of OA is 5:4



Given that the arc AB is 11cm  
Calculate the size of the shaded region  
Give your answer to 1 decimal place

**[6Marks]**

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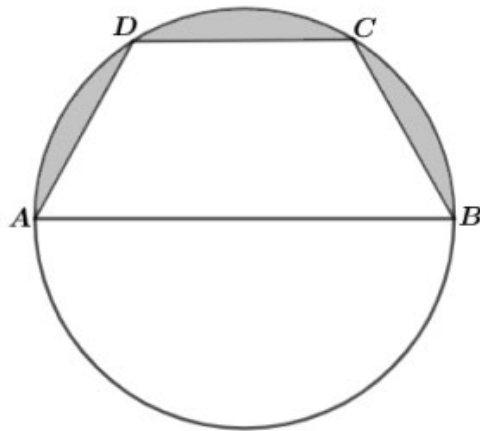
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- 3 An isosceles trapezium is drawn inside a circle of circumference 75cm where  $AD = BC = 14.5\text{cm}$   
The diagonal  $AC = 20.7\text{cm}$  and angle  $CBA = 60^\circ$



Calculate the shaded area correct to 1 decimal place **[6Marks]**

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4 The first four terms of a sequence are given below

$$3x, 2y, 3x + 2y, 3x + 5y, \dots$$

Given that the third and fourth terms of the sequence are 15 and 31 respectively, find the values of  $x$  and  $y$  **[4Marks]**

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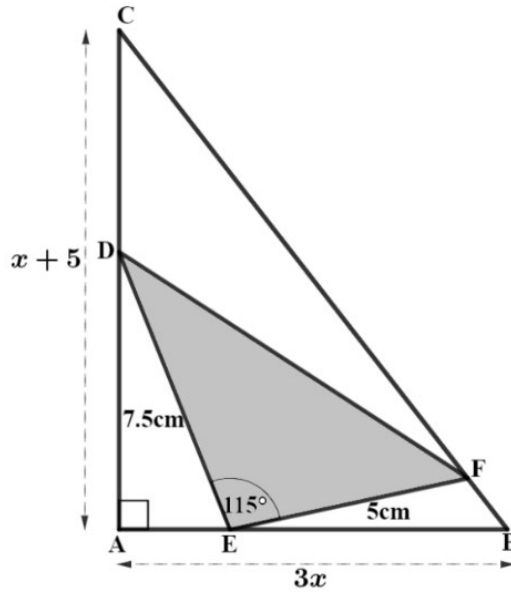
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5 Triangle ABC shown below is right-angled at the vertex A.



$$AC = x + 5$$

$$AB = 3x$$

$$ED = 7.5\text{cm}$$

$$EF = 5\text{cm}$$

$$\text{Angle DEF} = 115^\circ$$

Given that 45% of triangle ABC is shaded,  
find the value of  $x$  to 1 decimal place

[5Marks]

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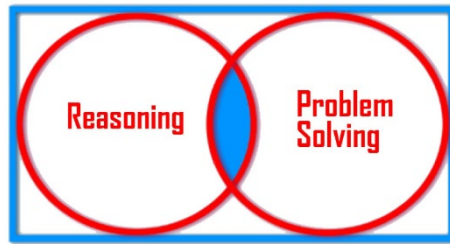
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A series of 30 horizontal lines for writing, arranged in a grid with a vertical margin line on the left.

# GCSE Mathematics (Grade 9-1)

## Problem Solving – Sample Questions (Solutions)

*H*



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{Aimed at students working towards a Grade 9 or 8}

**The grades and marks shown are for guidance purposes only**

**The questions are repeated here for your convenience**



- 1 Given that,  $\frac{x}{4} \div 1\frac{2}{3} = \frac{4}{3} - \frac{2}{x}$   
Find the possible values of  $x$   
Give your answers to 3 significant figures

**Solution**

Convert the mixed number to an improper fraction:  $1\frac{2}{3} = \frac{5}{3}$

Simplify the fractions on the LHS:

$$\frac{x}{4} \div 1\frac{2}{3} = \frac{x}{4} \div \frac{5}{3}$$

$$= \frac{x}{4} \times \frac{3}{5} = \frac{3x}{20}$$

[1mark]

Simplify the fractions on the RHS:

$$\frac{4}{3} - \frac{2}{x} = \frac{4x-6}{3x}$$

Therefore,  $\frac{3x}{20} = \frac{4x-6}{3x}$  {Cross multiply}

[1mark]

$$9x^2 = 80x - 120 \quad \{\text{Rearrange}\}$$

$$9x^2 - 80x + 120 = 0$$

[1mark]

Solve using the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

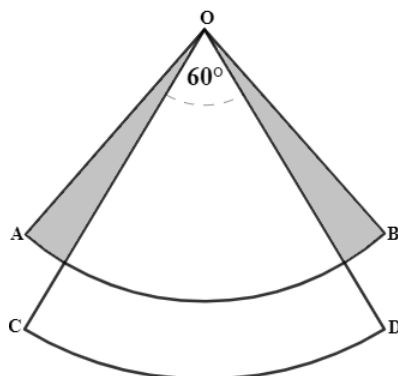
$$a = 9, b = -80 \text{ and } c = 120$$

$$x = \frac{-(-80) \pm \sqrt{(-80)^2 - 4 \times 9 \times 120}}{2 \times 9}$$

$$x = 6.98(3\text{sf}) \text{ or } x = 1.91(3\text{sf})$$

[2marks]

- 2 The sector OCD was formed from a circle with equation  $x^2 + y^2 = 64$   
The ratio of the length of OC to the length of OA = 5:4



Given that the arc AB is 11cm. Calculate the size of the shaded region.  
Give your answer to 1 decimal place.

**Solution**

The radius of the sector OCD can be found from the given equation of the circle as follows:  $r^2 = 64$  therefore,  $r = 8$ cm [1mark]

Therefore, the radius OA =  $\frac{4 \times 8}{5} = 6.4$ cm [1mark]

{Arc length =  $\frac{\theta}{360} \times 2\pi r$ } where  $\theta$  is the interior angle of the sector}

Therefore,  $11 = \frac{\theta}{360} \times \pi \times 2 \times 6.4$

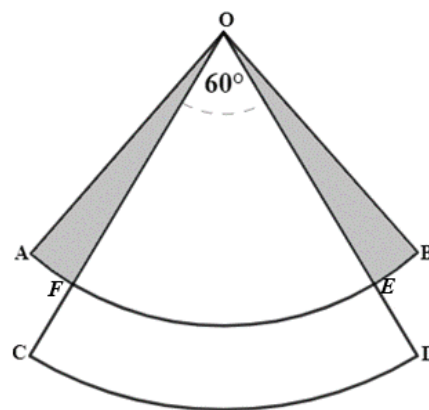
$\theta = \frac{11 \times 360}{2 \times 6.4 \times \pi} = 98.5^\circ$ (1dp) [1mark]

Hence, area of the sector AOB =  $\frac{98.5}{360} \times \pi \times 6.4^2 = 35.2$ (1dp) [1mark]

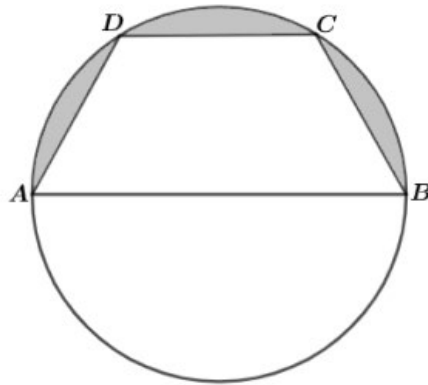
Now notice that the part of the sector OCD which is inside AOB will have the same radius as AOB. This is shown as OE and OF in the diagram on the RHS.

Therefore, Area of the sector EOF  
=  $\frac{60}{360} \times \pi \times 6.4^2 = 21.4$ (1dp) [1mark]

Hence, area of the shaded region  
 $35.2 - 21.4 = 13.8$ cm<sup>2</sup>(1dp) [1mark]



- 3 An isosceles trapezium is drawn inside a circle of circumference 75cm. AD = BC = 14.5cm. The diagonal AC = 20.7cm and angle CBA = 60°



Calculate the shaded area correct to 1 decimal place.

**Solution**

$$\{\text{Circumference} = \pi d\}$$

$$\text{Therefore, } 75 = \pi \times d$$

$$d = \frac{75}{\pi} = 23.9(1\text{dp})$$

$$\text{Therefore, } AB = 23.9\text{cm}$$

[1mark]

Hence, the radius of the circle will be  $23.9 \div 2 = 11.95\text{cm}$

Therefore, area of the semi-circle in which the trapezium is drawn will be

$$\frac{\pi \times (11.95)^2}{2} = 224.3(1\text{pd}) \quad [1\text{mark}]$$

Area of triangle ABC

$$= \frac{1}{2} \times 14.5 \times 23.9 \times \sin 60 = 150.1(1\text{dp})$$

[1mark]

Find angle CAB:

$$\frac{\sin CAB}{14.5} = \frac{\sin 60}{20.7}$$

$$\sin CAB = \frac{14.5 \times \sin 60}{20.7} = 0.606636..$$

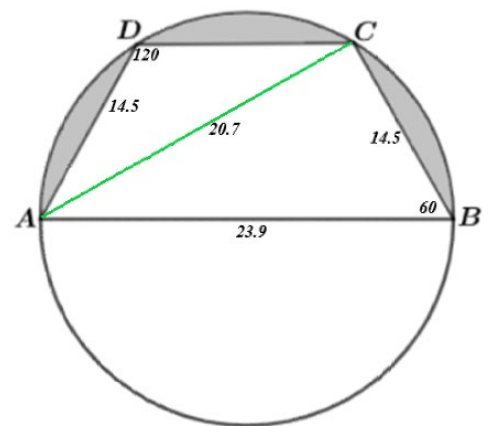
$$\text{Angle CAB} = \sin^{-1}(0.606636) = 37.3 \quad [1\text{mark}]$$

$$\text{Hence, angle CAD} = 60 - 37.3 = 22.7$$

$$\text{Area of triangle ACD} = \frac{1}{2} \times 14.5 \times 20.7 \times \sin 22.7 = 57.9 \quad [1\text{mark}]$$

$$\text{Hence, the area of the shaded region} = 224.3 - 150.1 - 57.9 = 16.3\text{cm}^2$$

[1mark]



4 The first four terms of a sequence are given below

$$3x, 2y, 3x + 2y, 3x + 5y, \dots$$

Given that the third and fourth terms of the sequence are 15 and 31 respectively, find the values of  $x$  and  $y$

**Solution**

Set up two simultaneous equations and solve to find  $x$  and  $y$

$$3x + 2y = 15 \text{ -----Equation 1}$$

$$3x + 5y = 31 \text{ -----Equation 2}$$

[1mark]

Subtract Equation 1 from Equation 2

$$3y = 16$$

$$y = \frac{16}{3}$$

[1mark]

Now substitute  $y = \frac{16}{3}$  into Equation 1

$$3x + 2\left(\frac{16}{3}\right) = 15$$

[1mark]

$$3x + \frac{32}{3} = 15$$

{Multiply through by 3}

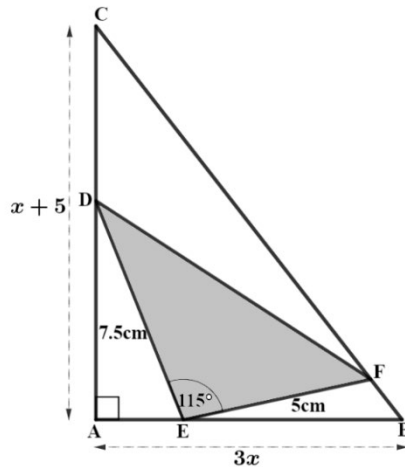
$$9x + 32 = 45$$

$$9x = 13$$

$$x = \frac{13}{9}$$

[1mark]

- 5 Triangle ABC shown below is right-angled at the vertex A.



$$AC = x + 5$$

$$AB = 3x$$

$$ED = 7.5\text{cm}$$

$$EF = 5\text{cm}$$

$$\text{Angle DEF} = 115^\circ$$

Given that 45% of triangle ABC is shaded,  
find the value of  $x$  to 1 decimal place

### Solution

Calculate the area of the shaded triangle, DEF using the sine rule:

$$\{\text{Area of a triangle} = \frac{1}{2} ab \times \sin C\}$$

$$\text{Therefore, Area of triangle DEF} = \frac{1}{2} \times 7.5 \times 5 \times \sin 115 = 16.9933$$

[1mark]

Let the area of triangle ABC be  $A$ . Then 45% of  $A = 16.9933$

$$\text{Therefore, } A = 16.9933 \div 45\% = 37.7629 \quad \text{[1mark]}$$

$$\text{Area of triangle ABC in terms of } x = \frac{1}{2} \times 3x \times (x + 5) = 1.5x^2 + 7.5x$$

$$\text{Therefore, } 1.5x^2 + 7.5x = 37.7629 \quad \{\text{Subtract } 37.7629 \text{ from both sides}\}$$

$$1.5x^2 + 7.5x - 37.7629 = 0 \quad \{\text{Divide through by } 1.5\} \quad \text{[1mark]}$$

$$\text{Solve using the quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1.5, \quad b = 7.5 \text{ and } c = -37.7629$$

$$x = \frac{-7.5 \pm \sqrt{(7.5)^2 - 4 \times 1.5 \times -37.7629}}{2 \times 1.5} \quad \text{[1mark]}$$

$$x = 3.1058(4\text{dp}) \text{ or } x = -8.1058(4\text{dp})$$

$$\text{Therefore, } x = 3.1(1\text{dp}) \quad \text{[1mark]}$$