Year 1 – AS

Sample Questions (Quadratics)

The marks shown are for guidance purposes only

Reasoning Problem Solving

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1 The roots of a quadratic function f(x), are given as $x = 3\sqrt{2}$ and $x = -\sqrt{2}$. Find the minimum point of the function. **[4Marks]**

2 Given that $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{8}{27}\right)^{-\frac{2}{3}+p}$, find the possible values of *p*.

Give your answers as exact values. **[5Marks]**

3 The function $f(x) = x^2 + Px + 7$, where P is a constant, intersects the function $g(x) = -2x^2 - 6x + 1$ at the point $(-3, 1)$ and at the point A. Find:

a) The value of P **[2Marks]**

b) The coordinates of the point A **[3Marks]**

c) Hence, find the minimum point of $f(x)$ **[2Marks]**

4 When the function $f(x) = x^2 + 6x + k$ is divided by $(x - a)$, the remainder is the same when $f(x)$ is divided by $(x - 2a)$

a) Find the possible values of a **[4Marks]**

b) The remainder when $f(x)$ is divided by $(x + 3a)$ is 5
Find the possible values of k **[3Marks]**

5 A function f is defined on a set of real numbers as

$f(x) = C - 2x - 4x^2$ where C is a constant

a) Given that the maximum value of f(x) is $\frac{21}{4}$, find the value of C **[5Marks]**

b) Hence determine whether there exists any point of contact between the line $y = 2x + 6$ and f(x) and find the coordinates of such a point if it exists. **[6Marks]**

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Sample Questions (Quadratics)
Solutions

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1	The roots of a quadratic function f(x) are given as $x = 3\sqrt{2}$ and $x = -\sqrt{2}$. Find the minimum point of the function. **<u>Solution</u>** If $x = 3\sqrt{2}$ and $x = -\sqrt{2}$ are the roots for f(x), then $f(x) = (x - 3\sqrt{2})(x + \sqrt{2})$ {Expand} **[1mark]** $f(x) = x^2 - (2\sqrt{2})x - 6$ {Now write in completed square form} **[1mark]** $f(x) = (x - \sqrt{2})^2 - 2 - 6$ {Simplify} $f(x) = (x - \sqrt{2})^2 - 8$ **[1mark]** Hence, the minimum point for f(x) is the coordinate $(\sqrt{2}, -8)$ **[1mark]**
2	Given that $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{8}{27}\right)^{-\frac{2}{3}+p}$, find the possible values of p Give your answers as exact values. **<u>Solution</u>** Simplify the fraction inside the brackets on the RHS $\left(\frac{8}{27}\right)^{-\frac{2}{3}+p} = \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}+p}$ **[1mark]** So we have $\left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}} = \left(\frac{2}{3}\right)^{3\times\left(-\frac{2}{3}+p\right)} = \left(\frac{2}{3}\right)^{-2+3p}$ **[1mark]** Therefore, $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{2}{3}\right)^{-2+3p}$ Notice that, the fractions in the brackets do not much, therefore, we need to apply the negative index law: $\{\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}\}$ Therefore, $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{3}{2}\right)^{2-3p}$ **[1mark]** Hence, $p^2 = 2 - 3p$ {Rearrange} $p^2 + 3p - 2 = 0$ **[1mark]** Solve using the quadratic solver in your calculator Therefore, $p = \frac{-3+\sqrt{17}}{2}$ or $p = \frac{-3-\sqrt{17}}{2}$ **[1mark]**

3	The function $f(x) = x^2 + Px + 7$ intersects the function $g(x) = -2x^2 - 6x + 1$ at the point $(-3, 1)$ and the point A Find: a) The value of P b) The coordinates of the point A c) Hence, find the minimum point of f(x) **<u>Solution</u>** a) To find the value of P, we need to substitute the given coordinate of intersection into f(x) When $x = -3, y = 1$ Therefore, we have, $1 = (-3)^2 + (-3)P + 7$ {Solve to find P} **[1mark]** $1 = -3P + 16$ $-3P = -15$ Hence, $P = 5$ **[1mark]** Therefore, $f(x) = x^2 + 5x + 7$ b) Now equate the two functions and solve to find the other point of intersection $x^2 + 5x + 7 = -2x^2 - 6x + 1$ {Simplify by rearranging} **[1mark]** $3x^2 + 11x + 6 = 0$ {Factorise} $(x + 3)(3x + 2) = 0$ Therefore, $x = -3$ or $x = -\frac{2}{3}$ **[1mark]** So, the x-coordinate of A will be $-\frac{2}{3}$ Substitute $x = -\frac{2}{3}$ into f(x) to find the y-coordinate of A Therefore, we have, $f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^2 + 5\left(-\frac{2}{3}\right) + 7 = \frac{37}{9}$ **[1mark]** Therefore, the coordinates of A will be $\left(-\frac{2}{3}, \frac{37}{9}\right)$ c) Write f(x) in completed square form $f(x) = x^2 + 5x + 7$ {Complete the square} $= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + 7$ {Simplify} **[1mark]** $= \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}$ Hence, the minimum point for f(x) is $\left(-\frac{5}{2}, \frac{3}{4}\right)$ **[1mark]**

4 When the function $f(x) = x^2 + 6x + k$ is divided by $(x - a)$, the remainder is the same as when $f(x)$ is divided by $(x - 2a)$

a) Find the possible values of a

b) The remainder when $f(x)$ is divided by $(x + 3a)$ is 5
Find the possible values of k

Solution

a) If $(x - a)$ leaves a remainder, then the remainder will be

$f(a) = (a)^2 + 6(a) + k$

$= a^2 + 6a + k$ **[1mark]**

Also, if $(x - 2a)$ leaves a remainder, then the remainder will be

$f(2a) = (2a)^2 + 6(2a) + k$

$= 4a^2 + 12a + k$ **[1mark]**

Since the two remainders are the same, it means that

$4a^2 + 12a + k = a^2 + 6a + k$ **[1mark]**

$3a^2 + 6a = 0$

$a^2 + 2a = 0$

Therefore, $a = 0$ or $a = -2$ **[1mark]**

b) If $(x + 3a)$ leaves a remainder, then the remainder will be

$f(-3a) = (-3a)^2 + 6(-3a) + k$

$= 9a^2 - 18a + k$

Therefore, $9a^2 - 18a + k = 5$ **[1mark]**

Using the possible values of 'a' obtained from **part a**, We have:

If $a = 0$ then $9(0)^2 - 18(0) + k = 5$

Therefore, $k = 5$ **[1mark]**

If $a = -2$ then $9(-2)^2 - 18(-2) + k = 5$

Therefore, $k = 67$ **[1mark]**

5	A function f is defined on a set of real numbers as $f(x) = C - 2x - 4x^2$ where C is a constant a) Given that the maximum value of f(x) is $\frac{21}{4}$, find the value of C b) Hence determine whether there exists any point of contact between the line $y = 2x + 6$ and f(x) and find the coordinates of such a point if it exists. **<u>Solution</u>** a) Write the function in completed square form $f(x) = -4x^2 - 2x + C$ {factorise -4 out} $= -4\left(x^2 + \frac{1}{2}x - \frac{C}{4}\right)$ **[1mark]** $= -4\left[\left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{C}{4}\right]$ {Simplify constant terms} **[1mark]** $= -4\left[\left(x + \frac{1}{4}\right)^2 - \frac{(4C+1)}{16}\right]$ {Multiply out the square brackets} $= -4\left(x + \frac{1}{4}\right)^2 + \frac{(4C+1)}{4}$ **[1mark]** Therefore, $\frac{21}{4} = \frac{4C+1}{4}$ **[1mark]** So $4C + 1 = 21$ $4C = 20$ Therefore, $C = 5$ **[1mark]** Hence, $f(x) = -4x^2 - 2x + 5$ b) Make the two equations equal $-4x^2 - 2x + 5 = 2x + 6$ {Rearrange} **[1mark]** $-4x^2 - 4x - 1 = 0$ **[1mark]** If $y = 2x + 6$ is a tangent to $f(x) = -4x^2 - 2x + 5$, then the discriminant $b^2 - 4ac = 0$ **[1mark]** $a = -4$ $b = -4$ $c = -1$ We have; $(-4)^2 - 4(-4)(-1) = 0$ Therefore, $y = 2x + 6$ is a tangent to $f(x) = -4x^2 - 2x + 5$ **[1mark]** Now we can find the point of contact by solving $-4x^2 - 4x - 1 = 0$ Factorise into $(2x + 1)^2 = 0$ therefore, $x = -\frac{1}{2}$ **[1mark]** If $x = -\frac{1}{2}$ then $y = 2\left(-\frac{1}{2}\right) + 6 = 5$ Hence, the point of contact is the coordinate $\left(-\frac{1}{2}, 5\right)$ **[1mark]**