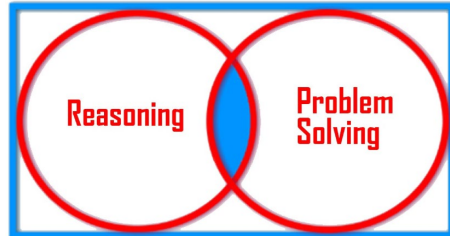


**Year 1 – AS**

**Sample Questions (Quadratics)**

**The marks shown are for guidance purposes only**



**fluidmaths.co.uk**







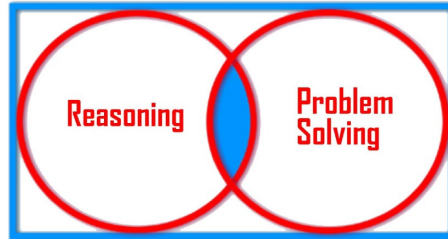






**Year 1 – AS**  
**Sample Questions (Quadratics)**  
**Solutions**

**The marks shown are for guidance purposes only**



**fluidmaths.co.uk**



**1** The roots of a quadratic function  $f(x)$  are given as  $x = 3\sqrt{2}$  and  $x = -\sqrt{2}$ . Find the minimum point of the function.

**Solution**

If  $x = 3\sqrt{2}$  and  $x = -\sqrt{2}$  are the roots for  $f(x)$ , then

$$f(x) = (x - 3\sqrt{2})(x + \sqrt{2}) \quad \{\text{Expand}\} \quad [1\text{mark}]$$

$$f(x) = x^2 - (2\sqrt{2})x - 6 \quad \{\text{Now write in completed square form}\} \quad [1\text{mark}]$$

$$f(x) = (x - \sqrt{2})^2 - 2 - 6 \quad \{\text{Simplify}\}$$

$$f(x) = (x - \sqrt{2})^2 - 8 \quad [1\text{mark}]$$

Hence, the minimum point for  $f(x)$  is the coordinate  $(\sqrt{2}, -8)$  [1mark]

**2** Given that  $\left(\frac{3}{2}\right)^{p^2} = \left(\frac{8}{27}\right)^{-\frac{2}{3}+p}$ , find the possible values of  $p$   
Give your answers as exact values.

**Solution**

Simplify the fraction inside the brackets on the RHS

$$\left(\frac{8}{27}\right)^{-\frac{2}{3}+p} = \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}+p} \quad [1\text{mark}]$$

$$\text{So we have } \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}} = \left(\frac{2}{3}\right)^{3 \times \left(-\frac{2}{3}+p\right)} = \left(\frac{2}{3}\right)^{-2+3p} \quad [1\text{mark}]$$

$$\text{Therefore, } \left(\frac{3}{2}\right)^{p^2} = \left(\frac{2}{3}\right)^{-2+3p}$$

Notice that, the fractions in the brackets do not match, therefore, we need to apply the negative index law:  $\left\{\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^c\right\}$

$$\text{Therefore, } \left(\frac{3}{2}\right)^{p^2} = \left(\frac{3}{2}\right)^{2-3p} \quad [1\text{mark}]$$

Hence,  $p^2 = 2 - 3p$  {Rearrange}

$$p^2 + 3p - 2 = 0 \quad [1\text{mark}]$$

Solve using the quadratic solver in your calculator

$$\text{Therefore, } p = \frac{-3+\sqrt{17}}{2} \text{ or } p = \frac{-3-\sqrt{17}}{2} \quad [1\text{mark}]$$

**3** The function  $f(x) = x^2 + Px + 7$  intersects the function  $g(x) = -2x^2 - 6x + 1$  at the point  $(-3, 1)$  and the point A  
Find:

- The value of P
- The coordinates of the point A
- Hence, find the minimum point of  $f(x)$

**Solution**

- To find the value of P, we need to substitute the given coordinate of intersection into  $f(x)$

When  $x = -3, y = 1$

Therefore, we have,  $1 = (-3)^2 + (-3)P + 7$  {Solve to find P} **[1mark]**

$$1 = -3P + 16$$

$$-3P = -15$$

Hence,  $P = 5$  **[1mark]**

Therefore,  $f(x) = x^2 + 5x + 7$

- Now equate the two functions and solve to find the other point of intersection

$$x^2 + 5x + 7 = -2x^2 - 6x + 1 \quad \{\text{Simplify by rearranging}\} \quad \mathbf{[1mark]}$$

$$3x^2 + 11x + 6 = 0 \quad \{\text{Factorise}\}$$

$$(x + 3)(3x + 2) = 0$$

Therefore,  $x = -3$  or  $x = -\frac{2}{3}$  **[1mark]**

So, the  $x$ -coordinate of A will be  $-\frac{2}{3}$

Substitute  $x = -\frac{2}{3}$  into  $f(x)$  to find the  $y$ -coordinate of A

$$\text{Therefore, we have, } f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^2 + 5\left(-\frac{2}{3}\right) + 7 = \frac{37}{9} \quad \mathbf{[1mark]}$$

Therefore, the coordinates of A will be  $\left(-\frac{2}{3}, \frac{37}{9}\right)$

- Write  $f(x)$  in completed square form

$$f(x) = x^2 + 5x + 7 \quad \{\text{Complete the square}\}$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + 7 \quad \{\text{Simplify}\} \quad \mathbf{[1mark]}$$

$$= \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}$$

Hence, the minimum point for  $f(x)$  is  $\left(-\frac{5}{2}, \frac{3}{4}\right)$  **[1mark]**

4 When the function  $f(x) = x^2 + 6x + k$  is divided by  $(x - a)$ , the remainder is the same as when  $f(x)$  is divided by  $(x - 2a)$

a) Find the possible values of  $a$

b) The remainder when  $f(x)$  is divided by  $(x + 3a)$  is 5

Find the possible values of  $k$

**Solution**

a) If  $(x - a)$  leaves a remainder, then the remainder will be

$$f(a) = (a)^2 + 6(a) + k$$

$$= a^2 + 6a + k \quad \text{[1mark]}$$

Also, if  $(x - 2a)$  leaves a remainder, then the remainder will be

$$f(2a) = (2a)^2 + 6(2a) + k$$

$$= 4a^2 + 12a + k \quad \text{[1mark]}$$

Since the two remainders are the same, it means that

$$4a^2 + 12a + k = a^2 + 6a + k \quad \text{[1mark]}$$

$$3a^2 + 6a = 0$$

$$a^2 + 2a = 0$$

$$\text{Therefore, } a = 0 \text{ or } a = -2 \quad \text{[1mark]}$$

b) If  $(x + 3a)$  leaves a remainder, then the remainder will be

$$f(-3a) = (-3a)^2 + 6(-3a) + k$$

$$= 9a^2 - 18a + k$$

$$\text{Therefore, } 9a^2 - 18a + k = 5 \quad \text{[1mark]}$$

Using the possible values of 'a' obtained from **part a**, We have:

$$\text{If } a = 0 \text{ then } 9(0)^2 - 18(0) + k = 5$$

$$\text{Therefore, } k = 5 \quad \text{[1mark]}$$

$$\text{If } a = -2 \text{ then } 9(-2)^2 - 18(-2) + k = 5$$

$$\text{Therefore, } k = 67 \quad \text{[1mark]}$$

**5** A function  $f$  is defined on a set of real numbers as  $f(x) = C - 2x - 4x^2$  where  $C$  is a constant

- a) Given that the maximum value of  $f(x)$  is  $\frac{21}{4}$ , find the value of  $C$   
b) Hence determine whether there exists any point of contact between the line  $y = 2x + 6$  and  $f(x)$  and find the coordinates of such a point if it exists.

**Solution**

- a) Write the function in completed square form

$$f(x) = -4x^2 - 2x + C \text{ \{factorise } -4 \text{ out\}}$$

$$= -4 \left( x^2 + \frac{1}{2}x - \frac{C}{4} \right) \quad \text{[1mark]}$$

$$= -4 \left[ \left( x + \frac{1}{4} \right)^2 - \frac{1}{16} - \frac{C}{4} \right] \quad \text{\{Simplify constant terms\} \quad \text{[1mark]}}$$

$$= -4 \left[ \left( x + \frac{1}{4} \right)^2 - \frac{(4C+1)}{16} \right] \quad \text{\{Multiply out the square brackets\}}$$

$$= -4 \left( x + \frac{1}{4} \right)^2 + \frac{(4C+1)}{4} \quad \text{[1mark]}$$

$$\text{Therefore, } \frac{21}{4} = \frac{4C+1}{4} \quad \text{[1mark]}$$

$$\text{So } 4C + 1 = 21$$

$$4C = 20 \quad \text{Therefore, } C = 5 \quad \text{[1mark]}$$

$$\text{Hence, } f(x) = -4x^2 - 2x + 5$$

- b) Make the two equations equal

$$-4x^2 - 2x + 5 = 2x + 6 \text{ \{Rearrange\} \quad \text{[1mark]}}$$

$$-4x^2 - 4x - 1 = 0 \quad \text{[1mark]}$$

If  $y = 2x + 6$  is a tangent to  $f(x) = -4x^2 - 2x + 5$ , then the discriminant  $b^2 - 4ac = 0$  [1mark]

$$a = -4$$

$$b = -4$$

$$c = -1$$

$$\text{We have; } (-4)^2 - 4(-4)(-1) = 0$$

Therefore,  $y = 2x + 6$  is a tangent to  $f(x) = -4x^2 - 2x + 5$  [1mark]

Now we can find the point of contact by solving  $-4x^2 - 4x - 1 = 0$

$$\text{Factorise into } (2x + 1)^2 = 0 \text{ therefore, } x = -\frac{1}{2} \quad \text{[1mark]}$$

$$\text{If } x = -\frac{1}{2} \text{ then } y = 2 \left( -\frac{1}{2} \right) + 6 = 5$$

Hence, the point of contact is the coordinate  $\left( -\frac{1}{2}, 5 \right)$  [1mark]