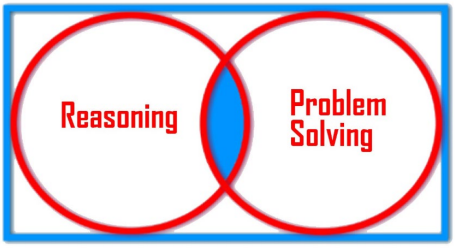
Year 1 – AS

Sample Questions (Exponentials and Logarithms)

The Marks shown are for guidance purposes only



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Round all decimals to 3 significant figures where applicable

1	Given that $\log x = a$ and $\log y = b$ Express $\log\left(\frac{10x^3}{\sqrt{y}}\right)$ in terms of a and b **[4Marks]**

2 Given that $\log x = (p + q)$ and $\log y = (p - q)$, express

a) $\log(xy)^3$ in terms of p **[4Marks]**

b) $\log\left(\frac{x}{100y}\right)$ in terms of q **[5Marks]**

3	Given that $y = 5 + 2^{5x+1}$, find y in terms of x **[5Marks]**

4	Given that $\log 2a + 2\log(b+1) = \log(a+1)$, a) Express b in terms of a **[5Marks]** b) Determine the possible values of b when $a = 8$ **[2Marks]**

5 A The pressure P and the volume V of a gas are related by the formula $P = kV^n$ where, k and n are constants
The information in the table below was obtained in an experiment in which P and V were measured

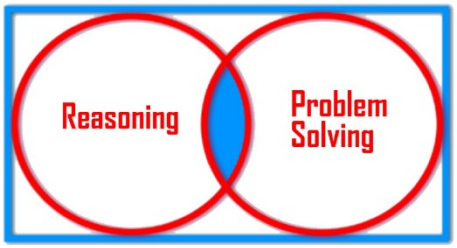
V	200	150	100	80	50
P	76	115	200	280	530

a) Express $P = kV^n$ in the form $y = mx + c$ **[2Marks]**

b) Find the values of k and n **[4Marks]**

Year 1 – AS

Sample Questions (Exponentials and Logarithms) (Solutions)



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The questions are repeated here for your convenience

1 Given that $\log x = a$ and $\log y = b$

Express $\log\left(\frac{10x^3}{\sqrt{y}}\right)$ in terms of a and b

Solution

$\log\frac{10a^3}{\sqrt{y}} = \log 10x^3 - \log\sqrt{y}$

{Applying $\log\left(\frac{a}{b}\right) = \log a - \log b$} **[1mark]**

$= \log_{10} 10 + \log_{10} x^3 - \log_{10} y^{\frac{1}{2}}$ **[1mark]**

{Applying $\log(a)^b = b\log a$}

$= 1 + 3\log_{10} x + \frac{1}{2}\log_{10} y$ **[1mark]**

{Applying $\log(ab) = \log a + \log b$}

Now sub in the values of $\log x$ and $\log y$

Therefore, we have

$= 1 + 3a + \frac{1}{2}b$ **[1mark]**

2 Given that $\log x = (p + q)$ and $\log y = (p - q)$, express

a) $\log x^3y^3$ in terms of p

b) $\log\left(\frac{x}{100y}\right)$ in terms of q

Solution

a) $\log x^3y^3 = \log x^3 + \log y^3$ **[1mark]**

$= 3\log x + 3\log y$ {Now sub in the values of $\log x$ and $\log y$}

$= 3(p + q) + 3(p - q)$ {Expand the brackets and simplify} **[1mark]**

$= 3p + 3q + 3p - 3q$ **[1mark]**

Therefore, $\log x^3y^3 = 6p$ **[1mark]**

b) $\log\left(\frac{x}{100y}\right) = \log x - \log 100y$ **[1mark]**

$= \log x - (\log 100 + \log y)$ {write 100 as powers of 10}

$= \log x - (\log 10^2 + \log y)$ **[1mark]**

$= \log x - (2\log 10 + \log y)$

$= \log x - 2 - \log y$ {Note that $\log 10 = 1$} **[1mark]**

{Now sub in the values of $\log x$ and $\log y$}

So, we have $\log\left(\frac{x}{100y}\right) = (p + q) - 2 - (p - q)$ {Simplify}

$= p + q - 2 - p + q$ **[1mark]**

Hence, $\log\left(\frac{x}{100y}\right) = 2q - 2$ **[1mark]**

3 Given that $y = 5 + 2^{5x+1}$, find y in terms of x

Solution

$y = 5 + 2^{5x+1}$

$y - 5 = 2^{5x+1}$ {Apply log to both sides of the equation}

$\log(y-5) = \log 2^{5x+1}$ **[1mark]**

$\log(y-5) = (5x+1)\log 2$ {Expand the brackets on the RHS} **[1mark]**

$\log(y-5) = 5x\log 2 + \log 2$ {Subtract $\log 2$ from both sides} **[1mark]**

$\log(y-5) - \log 2 = 5x\log 2$ {Divide both sides by $5\log 2$} **[1mark]**

Hence, $x = \frac{\log(y-5)-\log 2}{5\log 2}$ **[1mark]**

4 Given that $\log 2a + 2\log(b+1) = \log(a+1)$,

a) Express b in terms of a

b) Determine the possible values of b when $a = 8$

Solution

a) $\log 2a + 2\log(b+1) = \log(a+1)$

{Subtract $\log 2a$ from both sides}

$2\log(b+1) = \log(a+1) - \log 2a$ **[1mark]**

$\log(b+1)^2 = \log\left(\frac{a+1}{2a}\right)$ **[1mark]**

Apply the antilog to both sides

Therefore, $(b+1)^2 = \frac{a+1}{2a}$ {Take the square root on both sides} **[1mark]**

$b + 1 = \sqrt{\frac{a+1}{2a}}$ **[1mark]**

Hence, $b = \sqrt{\frac{a+1}{2a}} - 1$ **[1mark]**

b) $b = \sqrt{\frac{8+1}{2\times 8}} - 1$

$= \sqrt{\frac{9}{16}} - 1$

Therefore, $b = \frac{3}{4} - 1 = -\frac{1}{4}$ **[1mark]**

or $b = -\frac{3}{4} - 1 = -\frac{7}{4}$ **[1mark]**

5 The pressure P and the volume V of a gas are related by the formula $P = kV^n$ where, *k* and *n* are constants
The information in the table below was obtained in an experiment in which P and V were measured

V	200	150	100	80	50
P	76	115	200	280	530

a) Express $P = kV^n$ in the form $y = mx + c$
b) Find the values of *k* and *n*

Solution

a) $P = kV^n$ {Apply log to both sides}
$\log P = \log(kV^n)$ **[1mark]**
$\log P = \log k + n \log V$ {Rearrange in the form $y = mx + c$}
Hence, $\log P = n \log V + \log k$ **[1mark]**

b) Use corresponding values of V and P from the table
When $P = 76; V = 200$
Therefore, $76 = k(200^n) - -$Equation 1 **[1mark]**
When $P = 115; V = 150$
Therefore, $115 = k(150^n) - -$Equation 2 **[1mark]**

Divide Equation 1 by equation 2
$\frac{76}{115} = \frac{k(200)^n}{k(150)^n}$ {Cancel out the common factors}
$\frac{76}{115} = \left(\frac{4}{3}\right)^n$ {Apply log to both sides}
$\log\left(\frac{76}{115}\right) = n \log\left(\frac{4}{3}\right)$

Therefore, $n = \frac{\log\left(\frac{76}{115}\right)}{\log\left(\frac{4}{3}\right)} = -1.44(3sf)$ **[1mark]**

Now sub $n = -1.44$ into Equation 1
$76 = k(200^{-1.44})$
$k = \frac{76}{200^{-1.44}} = 15600(3sf)$ **[1mark]**
Hence $P = 15600(V)^{-1.44}$