Year 1 (AS)

Binomial Expansion
(Sample Questions Solutions)

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The questions are repeated here for your convenience

1	Write the expression $\left(\sqrt{5}+2\sqrt{7}\right)^5$ in the form $p\sqrt{5}+q\sqrt{7}$ where p and q are whole numbers to be found **<u>Solution</u>** $\left(\sqrt{5}+2\sqrt{7}\right)^5 = \binom{5}{0}\left(\sqrt{5}\right)^5 + \binom{5}{1}\left(\sqrt{5}\right)^4 2\sqrt{7} + \binom{5}{2}\left(\sqrt{5}\right)^3\left(2\sqrt{7}\right)^2 +$ $\binom{5}{3}\left(\sqrt{5}\right)^2\left(2\sqrt{7}\right)^3 + \binom{5}{4}\left(\sqrt{5}\right)\left(2\sqrt{7}\right)^4 + \binom{5}{5}\left(2\sqrt{7}\right)^5$ **[2Marks]** $= \left(25\sqrt{5}\right) + \left(250\sqrt{7}\right) + \left(1400\sqrt{5}\right) + \left(2800\sqrt{7}\right) + \left(3920\sqrt{5}\right) + \left(1568\sqrt{7}\right)$ Therefore, $\left(\sqrt{5}+2\sqrt{7}\right)^5 = \left(5345\sqrt{5}\right) + \left(4918\sqrt{7}\right)$ Where $p = 5345$ and $q = 4918$ **[2Marks]**
2	Show that ${}^{n}C_3 - {}^{n}C_2 = \frac{1}{6}(n^3 - 6n^3 + 5n)$ **<u>Solution</u>** Apply ${}^{n}C_r = \frac{n!}{r!(n-r)!}$ Therefore, ${}^{n}C_3 - {}^{n}C_2 = \frac{n!}{3!(n-3)!} - \frac{n!}{2!(n-2)!}$ {Expand each expression} $= \frac{n(n-1)(n-2)(n-3)!}{3!(n-3)!} - \frac{n(n-1)(n-2)!}{2!(n-2)!}$ {Cancel out the common factors} **[1mark]** $= \frac{n(n-1)(n-2)}{3\times 2} - \frac{n(n-1)}{2}$ $= \frac{n^3-3n^2+2n}{6} - \frac{3n^2-3n}{6}$ **[1mark]** $= \frac{n^3-6n^3+5n}{6}$ {factor out $\frac{1}{6}$} **[1mark]** $= \frac{1}{6}(n^3 - 6n^3 + 5n)$ As required **[1mark]**

3	The coefficient of x^3 in the expansion of the expression $(5+px)^7$ is 7503125. Find the value of p **<u>Solution</u>** Since the coefficient of x^3 is given as 7503125 So, $r = 3$ Therefore, we have $\binom{7}{3} \times 5^4 \times p^3 = 7503125$ **[1mark]** $21875p^3 = 7503125$ $p^3 = \frac{7503125}{21875}$ **[1mark]** $p^3 = 343$ $p = (343)^{\frac{1}{3}}$ $p = 7$ **[1mark]**

4	Find the coefficient of the term in x^5 in the expansion of the expression $(2+5x)(1+2x)^8$ **Solution** General term for $(1+2x)^8$ is ${}^{n}C_r = \binom{8}{r}(1)^{8-r}(2x)^{r}$ The term containing x^4 can be found as follows $\binom{8}{4} \times 1^4 \times (2x)^4$ $70 \times 2^4 \times x^4$ $1120 \times x^4$ **[1mark]** When this is multiplied by $5x$ from the first bracket, we will obtain a term in x^5 Also, the term containing x^5 can be found as follows $\binom{8}{5} \times 1^3 \times (2x)^5$ $56 \times 2^5 \times x^5$ $1792x^5$ **[1mark]** When this is multiplied by 2 from the first bracket, we will obtain another term in x^5 Therefore for the terms in x^5, we will have $= (2+5x)(1120x^4 + 1792x^5)$ $= 2 \times 1792x^5 + 5 \times 1120x^5$ $= 3584x^5 + 5600x^5$ $= 9184x^5$ **[1mark]** Hence, the required coefficient is 9184

5	Given that $(k+3x)^n = 16384 + 86016x + 193536x^2 + \cdots$ Where k and n are real numbers. Find the values of k and n **<u>Solution</u>** $(k+3x)^n = k^n + \frac{nk^{n-1}(3x)^1}{1!} + \frac{n(n-1)k^{n-2}(3x)^2}{2!}$ **[1mark]** By comparing the coefficients of x $(k+3x)^n = 16384 + 86016x + 193536x^2 + \cdots$ $k^n = 16384$----------------------------------A $3nk^{n-1} = 86016$--------------------------B $\frac{9}{2}n(n-1)k^{n-2} = 193536$-------------C **[1mark]** From B, $3n\frac{k^n}{k} = 86016$ {Applying division of indices} Sub, $k^n = 16384$ from A, $\frac{3n}{k}16384 = 86016$ $\frac{n}{k} = 1.75$ **[1mark]** Sub $k^n = 16384$ from A into to equation C $\frac{9}{2}n(n-1)\frac{16384}{k^2} = 193536$ **[1mark]** $n(n-1)\frac{1}{k^2} = 2.625$ Since $\frac{n}{k} = 1.75$ then $n = 1.75k$ **[1mark]** $\frac{1.75k(1.75k-1)}{k^2} = 2.625$ **[1mark]** $3.0625k - 1.75 = 2.625k$ $0.4375k = 1.75$ $k = 4$ **[1mark]** $n = 1.75k$ $n = 1.75(4)$ $n = 7$ **[1mark]**